

Name: \_\_\_\_\_

MATH 1050 - Exam 4 (Released) - 2012-13

- Neatly write your solutions directly on the exam paper. If a solution requires more space than given, you may continue on the back of the page. Work on scratch paper will not be graded.
- *To receive full credit you must show all necessary work and provide clear explanations.*
- Books, notes, calculators with symbolic manipulation features, computers, cell phones, or other internet enabled devices are not allowed.

1. Each of these problems is worth 3 points. NO justification is required for these problems.

(a) State whether the equation  $x^2 + 4y^2 - 8y = 0$  represents the equation of a circle, ellipse, parabola, or hyperbola.

(b) For the matrices  $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 4 \end{bmatrix}$ , find  $2A - B$ , if possible.

(c) Express the sum  $4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots + 8 \left(\frac{1}{2}\right)^n$  using summation notation. (Do not find the sum.)

$$\sum$$

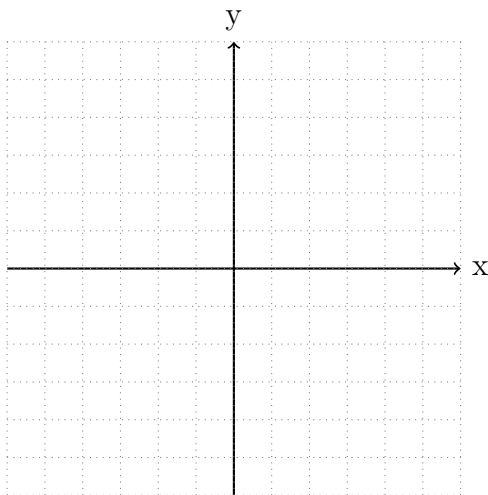
(d) Find the first 3 terms of the geometric sequence with  $r = \frac{1}{2}$  and  $a_1 = 5$ .

(e) For the arithmetic sequence  $100, 97, 94, 91, \dots$ , find  $d$ , the common difference.

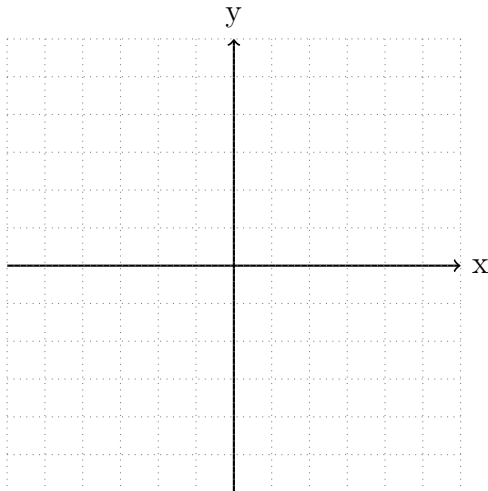
(f) Find a formula for the  $n^{\text{th}}$  term of the sequence  $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$ . Hint: this sequence is neither arithmetic nor geometric.

2. Write the hyperbola  $2x^2 - y^2 - 4x - 6y = 11$  in standard form.

3. A parabola has a vertex at  $(-2, 3)$  and a directrix at  $x = 0$ . Find the equation of the parabola. (You do not need to graph the parabola.)



4. Graph the ellipse  $(x - 1)^2 + 4(y + 2)^2 = 16$ . Label the center, vertices, and foci.



5. Find the first four terms of the sequence  $\{a_n\}$  defined recursively by  $a_1 = 2$ ,  $a_n = na_{n-1} - 3$

6. If  $\{a_n\}$  is an arithmetic sequence with  $a_6 = 40$  and  $a_{24} = 10$ , find  $d$ , the common difference.

7. Find the formula for the  $n^{\text{th}}$  term of the series  $24, 12, 6, 3, \dots$

8. Find the sum  $\sum_{k=1}^{20} (4k - 2)$ .

9. Determine whether the series  $\sum_{k=1}^{\infty} \frac{(-1)^k}{5}$  converges or diverges. If it converges, find its sum.

10. Solve the system using elimination: 
$$\left\{ \begin{array}{rcl} -x & + & y - 5z = -15 \\ x & + & 3y + z = -1 \\ & & y - z = -4 \end{array} \right\}$$

11. The matrix  $\begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$  represents an augmented matrix for a linear system. Write the corresponding set of linear equations.

12. Determine whether Cramer's rule can be used to solve the linear system:

$$\left\{ \begin{array}{rcl} 3x & - & 6y - z = 24 \\ -2x & + & 5y + 3z = -19 \\ x & - & y + z = 10 \end{array} \right\}$$

If it can't be used, explain why not. If Cramer's rule can be used, use Cramer's rule to solve for  $z$ . You may use a calculator to find determinants, but must show all other work.

13. Solve the linear system  $\begin{cases} 3x - 6y = 12 \\ -2x + 5y = -9 \end{cases}$  using the following method:

(a) Write the linear system as a matrix equation  $A\mathbf{X} = \mathbf{B}$ .

(b) Find  $A^{-1}$  using any method *AND* use the inverse matrix to solve the matrix equation for  $\mathbf{X}$ .

14. Find the inverse of the matrix  $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & -1 \end{bmatrix}$ . Show all work.