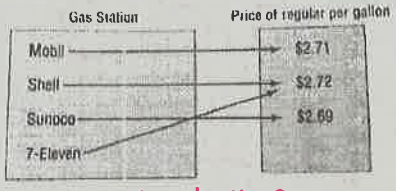
3.1 Functions

What makes something a function?

for every x there is exactly one y

Is this relation a function? If yes, list the domain and range.

Figure 7



yes

D: {Mobil, Shell, Sunoco, 7113}

R: {2.71, 2.72, 2.69}

Is $-2x + y = -5$ a function?

Get y by itself & plug in x=0

$y = -5 + 2x$ if $x=0$ then $y=-5$

Yes it's a function

Is $x^2 + y^2 = 4$ a function?

$y^2 = 4 - x^2$

$y = \pm\sqrt{4-x^2}$ if $x=0$ then

$y = \pm\sqrt{4-0^2}$

$y = \pm\sqrt{4}$

$y = \pm 2$

For every input, x, there's 2 outputs, y; so NOT A FUNCTION

Finding the value of a function.

$$f(x) = 2x^2 - 3x + 5$$

a) f(-4)
b) f(x) + f(-4)
c) 2f(x)
d) f(3x)
e) $\frac{f(x+h)-f(x)}{h}$ $h \neq 0$

a) $f(-4) = 2(-4)^2 - 3(-4) + 5$

$f(-4) = 32 + 12 + 5 = 49$ | $f(-4) = 49$

b) $f(x) + f(-4) = (2x^2 - 3x + 5) + (49)$

$f(x) + f(-4) = 2x^2 - 3x + 54$

c) $2f(x) = 2(2x^2 - 3x + 5)$

$2f(x) = 4x^2 - 6x + 10$

d) $f(3x) = 2(3x)^2 - 3(3x) + 5$

$f(3x) = 2(9x^2) - 9x + 5$

$f(3x) = 18x^2 - 9x + 5$

e) $\frac{f(x+h)-f(x)}{h}$

$\frac{[2(x+h)^2 - 3(x+h) + 5] - (2x^2 - 3x + 5)}{h}$

$(x+h)(x+h) = x^2 + 2xh + h^2$ so $2(x^2 + 2xh + h^2)$

$\frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 5 - 2x^2 + 3x - 5}{h}$

$\frac{4xh + 2h^2 - h}{h}$ factor out h $\frac{h(4x + 2h - 3)}{h}$

$\frac{f(x+h)-f(x)}{h} = 4x + 2h - 3$

(to find derivative plug in h=0)

$f(x) = x^3 - 4x$

$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - 4(x+h) - (x^3 - 4x)}{h}$

since $(x+h)(x+h)(x+h) = x^3 + 3x^2h + 3xh^2 + h^3$

$\frac{x^3 + 3x^2h + 3xh^2 + h^3 - 4x - 4h - x^3 + 4x}{h}$

$\frac{3x^2h + 3xh^2 + h^3 - 4h}{h}$

$\frac{h(3x^2 + 3xh + h^2 - 4)}{h}$

$\frac{f(x+h) - f(x)}{h} = 3x^2 + 3xh + h^2 - 4$

Domain

When do we have to worry about domain issues?

- When you have a denominator $\neq 0$
- $\sqrt{\ }$ radical with even index must be ≥ 0

Finding the Domain of a Function

Find the domain of each of the following functions:

(a) $f(x) = x^2 + 5x$ D: $\{x \mid x \in \mathbb{R}\}$

(b) $g(x) = \frac{3x}{x^2 - 4}$ D: $\{x \mid x \neq 2, -2\}$

(c) $h(t) = \sqrt{4 - 3t}$

$4 - 3t \geq 0$

$-3t \geq -4$

$t \leq 4/3$

D: $\{t \mid t \leq 4/3\}$

(d) $F(x) = \frac{\sqrt{3x + 12}}{x - 5}$

$3x + 12 \geq 0$

$3x \geq -12$

$x \geq -4$

$x - 5 \neq 0$

$x \neq 5$

D: $\{x \mid x \neq 5, \text{ and } x \geq -4\}$

Ex.

$f(x) = \frac{1}{x+2} \quad g(x) = \frac{x}{x-1}$

Find the following and determine the domain of each

a) (f+g)(x)

$\frac{1}{x+2}\cdot\frac{(x-1)}{(x-1)} + \frac{x}{x-1}\cdot\frac{(x+2)}{(x+2)} = \frac{x - 1 + x^2 + 2x}{(x+2)(x-1)}$

$= \frac{x^2 + 3x - 1}{(x+2)(x-1)}$ D: $\{x \mid x \neq -2, 1\}$

b) (f-g)(x)

$\frac{1}{x+2} - \frac{x}{x-1}$

$\frac{x - 1 - (x^2 + 2x)}{(x+2)(x-1)} = \frac{-x^2 - x - 1}{(x+2)(x-1)}$

D: $\{x \mid x \neq -2, 1\}$

c) (f·g)(x)

$\frac{1}{(x+2)} \cdot \frac{x}{(x-1)} = \frac{x}{(x+2)(x-1)}$

D: $\{x \mid x \neq -2, 1\}$

Notice all three (a, b, c) have same domain

d) (f/g)(x)

$\frac{\frac{1}{x+2}}{\frac{x}{x-1}}$ } Domain $\neq$ what ever makes any of these equal zero

$\frac{1}{x+2} \cdot \frac{x-1}{x} = \frac{x-1}{x(x+2)}$

D: $\{x \mid x \neq -2, 0\}$

e) (g/f)(x)

$\frac{\frac{x}{x-1}}{\frac{1}{x+2}}$ } Domain $\neq 1 \neq -2$

$\frac{x}{x-1} \cdot \frac{x+2}{1} = \frac{x(x+2)}{x-1}$

D: $\{x \mid x \neq 1, -2\}$

Domains of:

f + g
f - g
f·g
f/g

} D: $\{x \mid x = \text{Domain of } f \text{ and Domain } G\}$

f/g } D: $\{x \mid g(x) \neq 0$ and D: f and D G$\}$

↳ so set $g(x) = 0$ & $x \neq$ that #