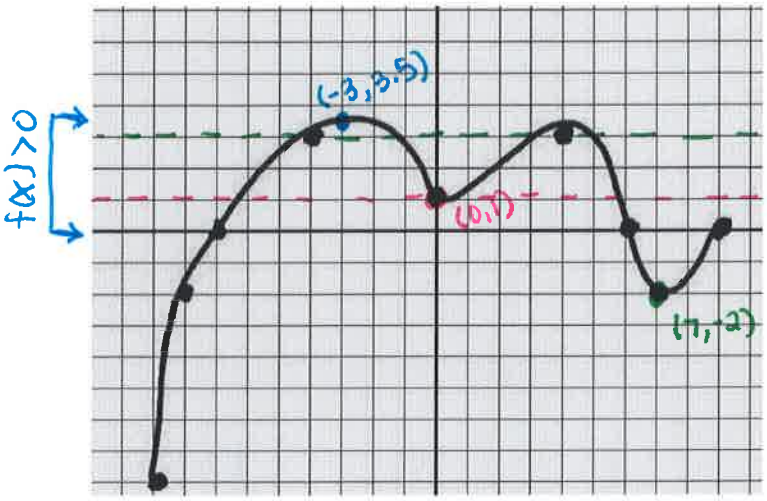
3.2 The Graph of Functions

What makes something a function?

for every x value there is exactly one y
*Use vertical line test

Example 1



a) Is this relation a function? yes

b) Find $f(-3), f(0)$, and $f(7)$

x=-3 f(0) x=7
y=3.5 x=0 y=-2
y=1

f(-3)=3.5 f(0)=1 f(7)=-2

c) What is the domain? D: $\{x \mid -9 \le x \le 9\}$

d) What is the range? R: $\{y \mid -8 \le y \le 3.5\}$

e) What are the intercepts (if any)?

x intercepts let y=0
so f(x)=0 so
x=-7, 6, 9
(-7,0), (6,0) (9,0)

y intercepts let x=0
f(0)=y y=1
(0,1)

f) How often does $y = 1$ intersect the graph?

@ three points

g) For what values of x does $f(x) = 3$? y=3

x=-4, -1, 3, 5 so {-4, -1, 3, 5}

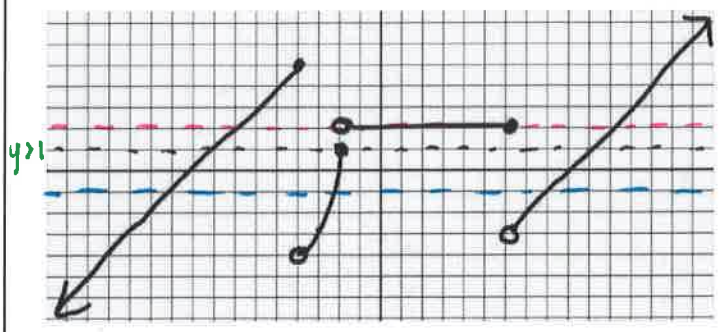
h) For what values is $f(x) > 0$? means y>0

$\{x \mid -7 < x < 6\}$

i) For what values is $f(x) \ge 0$? Now it can also be positive so

$\{x \mid -7 \le x \le 6 \cup x = 9\}$

Example 2



a) Is this relation a function? remember that the open circles doesn't include that pt so yes!

b) Find $f(-8), f(-4), f(0)$, and $f(5)$

when x=-8 when x=-4 x=0 x=5
y=1 y=5 y=2 y=2
f(-8)=1 f(-4)=5 f(0)=2 f(5)=2

c) What is the domain? D: $\{x \mid x \in \mathbb{R}\}$

d) What is the range? R: $\{y \mid y \in \mathbb{R}\}$

e) What are the intercepts (if any)?

y intercepts
when x=0
y=2
(0, 2)

x intercepts
when y=0
x=-9, -2, 9
(-9,0), (-2,0), (9,0)

f) How often does $y = -1$ intersect the graph?

@ 3 points

g) For what values of x does ~~f(x)=1? x<4~~
f(x)=1 so y=1
x=-8, x=-2, x=10
{-8, -2, 10}

h) For what values of x does $f(x) = 2$? y=2

x=-7 $-2 \le x \le 6$ x=11

$\{x \mid x = -7 \cup x = 11 \cup -2 < x \le 6\}$

i) For what values is $f(x) > 1$? y>1

$-8 < x \le 12$ $-2 < x \le 6$ x>10

$\{x \mid -8 < x \le 12 \cup -2 < x \le 6 \cup x > 10\}$

Example 3 $f(x) = x^3 - 2x^2 - 3x$

a) Is the point (4,24) on the graph of f?

x y

$(4)^3 - 2(4)^2 - 3(4) \stackrel{?}{=} 24$ so (4,24)

$64 - 32 - 12 \stackrel{?}{=} 24$ is NOT on

$20 \stackrel{?}{=} 24$ False the graph

b) If x=-1 what is the point on the graph of f?

$(-1)^3 - 2(-1)^2 - 3(-1) =$

$-1 - 2 + 3 = 0$

$(-1, 0)$

c) If ~~f(x)~~=-10, what are the point(s) on the graph of f?

$(-10)^3 - 2(-10)^2 - 3(-10) =$

$-1000 - 200 + 30 = -1170$

$f(-10) = -1170$

$(-10, -1170)$

d) What is the domain of the function f?

There are no $\sqrt{\ }$ (radicals w/ even index) or $\frac{}{}$ denominators so theres No domain restrictions so

$D: \{x \mid x \in \mathbb{R}\}$

e) List any x & y intercepts on the graph of f.

X intercepts
let y=0

$x^3 - 2x^2 - 3x = 0$

$x(x^2 - 2x - 3) = 0$

$x(x+1)(x-3) = 0$

$x = 0, -1, 3$ $\boxed{(0,0), (-1,0), (3,0)}$

Y intercepts
let x=0

$(0)^3 - 2(0)^2 - 3(0) =$

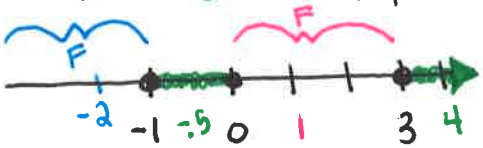
$0 - 0 - 0 = 0$

same $(0,0)$

f) When is $f(x) \geq 0$?

To find when $f(x) \geq 0$ set the equation equal to zero to find the X intercepts

we know the x intercepts are at x= 0, -1, 3 from part (e) if the graph was intercepts at these 3 pour point we need to see whether its above or below x axis (above is ≥0) so choose points in each region



$(-2)^3 - 2(-2)^2 - 3(-2) \geq 0$

$-8 - 8 + 6 \geq 0$

$-10 \geq 0$

False

$(-.5)^3 - 2(-.5)^2 - 3(-.5) \geq 0$

$-.125 + .5 + 1.5 \geq 0$

$.75 \geq 0$

T

$(1)^3 - 2(1)^2 - 3(1) \geq 0$

$1 - 2 - 3 \geq 0$

$-4 \geq 0$

F

$(4)^3 - 2(4)^2 - 3(4) \geq 0$

$64 - 32 - 12 \geq 0$

$20 \geq 0$ T

$\{x \mid -1 \leq x \leq 0 \text{ U } x \geq 3\}$