3.3 Properties of Functions

A function is even if: it's symmetric with the

y axis

To test algebraically you see if

$(x, y) \rightarrow (-x, y)$

$f(-x) = f(x)$

A function is odd if: it's symmetric with the

origin

To test algebraically you see if

$(x, y) \rightarrow (-x, -y)$

$f(-x) = -f(x)$

If a function isn't even or odd, then it's

Neither

(Functions can only be even or odd. Never both)

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How to test for Even/Odd Algebraically:

$f(x) = x^2 - 5$

even? if $f(x) = f(-x)$

$f(-x) = (-x)^2 - 5$

$= x^2 - 5$

since

$x^2 - 5 = x^2 - 5$ then $f(x) = f(-x)$

so it's $\boxed{\text{even}}$

$g(x) = x^3 - 1$

even?

$g(x) = g(-x)$

$= (-x)^3 - 1$

$= -x^3 - 1$

not even

odd?

$-g(x) = g(-x)$

$-(x^3 - 1) = -x^3 + 1$

since $-x^3 + 1 \neq -x^3 - 1$

its not odd

$\boxed{\text{Neither}}$

$h(x) = 5x^3 - x$

even?

$h(-x) = 5(-x)^3 - (-x)$

$= -5x^3 + x$

Not even

odd if $h(-x) = -h(x)$

$-h(x) = -(5x^3 - x)$

$= -5x^3 + x$

since $h(-x) = -h(x)$

its $\boxed{\text{odd}}$

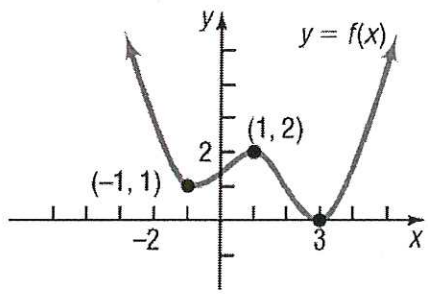
A function f is **increasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$. goes up from left to right

A function f is **decreasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) > f(x_2)$. goes down from Left to right

A function f is **constant** on an interval I if, for all choices of x in I, the values $f(x)$ are equal. is a horizontal line

The Local minimum value is/are f(c) or y value at the number(s) x= x value or c

The Local maximum value is/are f(c) or y value at the number(s) x= x value or c



(a) At what number(s), if any, does f have a local maximum?

There is a local max at $x=1$

(b) What are the local maxima?

the local max value is 2

(c) At what number(s), if any, does f have a local minimum?

There is a local min @ $x=-1$ & $x=3$

(d) What are the local minima?

The local min values are 1 & 0

(e) List the intervals on which f is increasing.

$(-1, 1) \cup (3, \infty)$

(f) List the intervals on which f is decreasing.

$(-\infty, 1) \cup (1, 3)$
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Using your "Graphing Utility" Domain interval in window

$f(x) = 2x^3 - 3x + 1 \quad (-2, 2)$

a) Find any local maximum/minimum

b) Determine where f is increasing/ decreasing

Increasing : (-2, -.

Average Rate of Change:

The average rate of change from a to b is defined as:

$$\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a} \quad b \neq a$$

Find the average rate of change of $f(x) = 3x^2$

A) From 1 to 3

$f(3) = 3(3)^2 = 27 \qquad f(1) = 3(1)^2 = 3$

$$\frac{\Delta y}{\Delta x} = \frac{27-3}{3-1} = \frac{24}{2} = \boxed{12}$$

B) From 1 to 7

$f(7) = 3(7)^2 = 147 \qquad f(1) = 3(1)^2 = 3$

$$\frac{\Delta y}{\Delta x} = \frac{147-3}{7-1} = \frac{144}{6} = \boxed{24}$$

Slope of the Secant Line:

The average rate of change of a function from a to b equals the slope of the secant line containing the two points (a, f(a)) and (b, f(b)) on its graph.

$g(x) = 3x^2 - 2x + 3$

a) Find the average rate of change of g from -2 to 1.

$$\frac{\Delta y}{\Delta x} = \frac{f(1)-f(-2)}{1+2} =$$

$$\frac{3(1)^2 - 2(1) + 3 - [3(-2)^2 - 2(-2) + 3]}{1+2}$$

$g(1)$

$$\frac{(3-2+3) + 12 - 4 - 3}{3} = \frac{9}{3} = \boxed{3}$$

b) Find an equation of the secant line containing (-2, g(-2)) and (1, g(1)).

point-slope form since $(1, g(1)) = (1, 4)$

$y - 4 = 3(x - 1)$

$y - 4 = 3x - 3$

$+4 \qquad +4$

$\boxed{y = 3x + 1}$

Difference Quotient: *the bridge between Algebra (slope) and Calculus (derivative)*

$$m_{sec} = \frac{f(x+h) - f(x)}{h} \qquad h \neq 0$$

so much fun!!