5.6- Complex Zeros; Fundamental Theorem of Algebra

Fundamental Theorem of Algebra:

Every Complex polynomial Function $f(x)$ of degree $n\geq 1$ has at least one complex zero.

You can factor every complex polynomial function of degree n into "n" factors factors.

Complex Zeros of a polynomial function means All Zeros!!

Conjugate Pairs Theorem:

Basically says: "If $a+bi$ is a real zero of f, then $a-bi$ is also a real zero of f.

** Make sure to distribute when writing these out!

Ex: $x=3i$ is one of the solutions of a polynomial of degree 2 Then its conjuate pair $x=-3i$ is also a real zero

$(x-3i)(x+3i)$ you could foil or you could do the trick

$x=\pm 3i$

$x^2=(\pm 3i)^2$

$x^2=-9$ $\boxed{x^2+9=0}$

Writing in "completely factored form" or "as a product of **LINEAR** factors.

A polynomial of degree 5 whose coefficients are real numbers has the zeros -2, $-3i$, and $2+4i$. Find the remaining two zeros.

Since the degree is 5 we need 5 zeros

Since two are complex $-3i$ & $2+4i$ we need their conjugate pairs as solution

the conjugate of $-3i$ is $3i$ & of $2+4i$ is $2-4i$

$x=3i,\ 2-4i$ are the missing solutions

Finding a polynomial from its zeros:

Find a polynomial f of degree 4 whose coefficients are real numbers and that has the zeros 1, 1, $-4+i$.

conjugate $-4-i$

$f(x)=a(x-1)(x-1)[x-(-4+i)][x-(-4-i)]$

you could foil the complex solutions or you could use the trick

$x=-4\pm i$
$+4\quad +4$

$x+4=\pm i$

$(x+4)^2=(\pm i)^2$

$x^2+8x+16=-1$

$x^2+8x+17=0$ Thus $[x-(-4+i)][x-(-4-i)]$ is equal to $x^2+8x+17$

$f(x)=a(x-1)(x-1)(x^2+8x+17)$

$=a(x^2-2x+1)(x^2+8x+17)$ Foil

$=a(x^4+8x^3+17x^2$
$\quad -2x^3-16x^2-34x$
$+\quad +x^2+8x+17)$

$=\boxed{a(x^4+6x^3+2x^2-26x+17)}$

Find the Complex Zeros of a Polynomial Function:

Find the complex zeros of the polynomial function and write f in factored form.

$f(x)=x^4+2x^3+x^2-8x-20$ 4 complex zeros

$\frac{p}{q}: \pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

To save time I'll pick $(x+2)$

$$\begin{array}{r|rrrrr} -2 & 1 & 2 & 1 & -8 & -20 \\ & \downarrow & -2 & 0 & -2 & 20 \\ \hline & 1 & 0 & 1 & -10 & \boxed{0} \end{array}$$

$(x+2)(x^3+x-10)$

pick $(x-2)$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 1 & -10 \\ & & 2 & 4 & 10 \\ \hline & 1 & 2 & 5 & \boxed{0} \end{array}$$

$(x+2)(x-2)(x^2+2x+5)$

~~Factor~~ solve

x^2+2x+5 $a=1$ $b=2$ $c=5$ plug into quadratic formula

$\frac{-2\pm\sqrt{(2)^2-4(1)(5)}}{2(1)}=\frac{-2\pm\sqrt{-16}}{2}=\boxed{-1\pm 2i}$

Zeros are $x=2,-2,-1+2i,-1-2i$

in factored form we have the polynomial

$\boxed{(x-2)(x+2)[x-(-1+2i)][x-(-1-2i)]}$

or $(x-2)(x+2)(x+1-2i)(x+1+2i)$

Use the given zero to find the complex zeros of
$f(x) = x^4 - 7x^3 + 14x^2 - 38x - 60$
And a zero exists at $x=1+3i$ use the trick

$x = 1 \pm 3i$
$-1 \quad -1$

$(x-1)^2 = (\pm 3i)^2 \qquad x^2 - 2x + 1 = -9$

$\boxed{x^2 - 2x + 10}$

Now use long division

$$\begin{array}{r} x^2 - 5x - 6 \\ x^2 - 2x + 10 \,\big)\, x^4 - 7x^3 + 14x^2 - 38x - 60 \\ -(x^4 - 2x^3 + 10x^2) \downarrow \\ \hline -5x^3 + 4x^2 - 38x \\ -(-5x^3 + 10x^2 + 50x) \downarrow \\ \hline -6x^2 + 12x - 60 \\ -(-6x^2 + 12x - 60) \\ \hline \boxed{0} \end{array}$$

Thus

$(x^2 - 2x + 10)(x^2 - 5x - 6)$

This factors into the answers from above

factor

$(x+1)(x-6)$

$\boxed{[x-(1+3i)][x-(1-3i)](x+1)(x-6)}$

saves you alot of time if you dont believe me use the other half of the pg using $\frac{p}{q}$ method.