6.1 Composition Functions $(f \circ g)(x) = f(g(x))$

Given two functions f and g, the **composite function**, denoted by $f \circ g$ (read as "f composed with g"), is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.

It is also read as f of g

Evaluating a composite function:

For $f(x) = 2x^2 + 3$ and $g(x) = 4x^3 + 1$ find:

a) $(f \circ g)(1)$

$g(1) = 4(1)^3 + 1$

$g(1) = 5$

$f(5) = 2(5)^2 + 3 = \boxed{53}$

b) $(g \circ f)(1)$

$f(1) = 2(1)^2 + 3 = 5$

$g(5) = 4(5)^3 + 1$

$500 + 1$

$\boxed{501}$

c) $(f \circ f)(-2)$

$2(-2)^2 + 3$

11

$2(11)^2 + 3$

$242 + 3$

$\boxed{245}$

d) $(g \circ g)(-1)$

$4(-1)^3 + 1$

-3

$4(-3)^3 + 1$ $\boxed{-107}$

* Graphing Calculators can also be used to Evaluate a composition function

Vars → Y vars

$(Y_1(Y_2(1))$

$Y_2(Y_2(-1))$

Steps to finding the Domain of a composite function (Use this method if you only need to find the domain not the composite function)

1. You must look for domain restrictions on the inside function and exclude the value(s) from the domain of the composition function
2. You must look for the domain restrictions on the outer function. Then set the inside function equal to the restriction just found and solve to find the second domain restriction.

Finding the Domain of $(f \circ g)(x)$ **if**

$f(x) = \dfrac{3}{x-5}$ and $g(x) = \dfrac{2}{x+1}$

not an answer yet because you have to plug in.

1. $g(x) = \dfrac{2}{x+1}$ $x \neq -1$

$f(x) = \dfrac{3}{x-5}$ $x \neq 5$ set equal to inside function

$(x+1)\dfrac{2}{x+1} = 5(x+1)$

$2 = 5x + 5$

$-3 = 5x$

$x \neq -3/5$

$\{x \mid x \neq -1, -3/5\}$

Finding a Composite Function and its Domain:

Suppose that $f(x)=\frac{3}{x-5}$ and $g(x)=\frac{2}{x+1}$

Find The Following:

a) $(f\circ g)(x)$

$g(x)=\frac{2}{x+1}$ $x\neq -1$

$f\left(\frac{2}{x+1}\right)=\frac{3}{\left(\frac{2}{x+1}\right)-5}\frac{(x+1)}{(x+1)}=\frac{3}{\frac{2-5x-5}{x+1}}$

$\frac{3}{1}\cdot\frac{x+1}{-5x+3}$ $=\frac{3x+3}{-5x-3}=0$

$-5x=3$ $x=-3/5$

b) $(f\circ f)(x)$

$\boxed{x\neq 5}$ Domain

$f\left(\frac{3}{x-5}\right)=\frac{3}{\left(\frac{3}{x-5}\right)-5}\frac{(x-5)}{(x-5)}=\frac{3}{\frac{3-5x+25}{x-5}}$

$\frac{3}{1}\cdot\frac{x-5}{-5x+28}$ $\boxed{\frac{3x-15}{-5x+28}}=0$ composition function

$-5x+28$

$-5x=-28$

$\boxed{x\neq 28/5}$ Domain

$\{x \mid x\neq 28/5, 5\}$

Show If Two Composite Functions are Equal:

If $(f\circ g)(x)=x$ and $(g\circ f)(x)=x$ then the composite functions are equal

Example: $f(x)=-2x+1,\ g(x)=-\frac{1}{2}(x-1)$

$f(g(x))$

$f\left(-\frac{1}{2}(x-1)\right)=-2\left(-\frac{1}{2}(x-1)\right)+1$

$-2\left(\frac{1}{2}x+1/2\right)+1=x-1+1=\boxed{x}$

$g(f(x))$

$g(-2x+1)=-\frac{1}{2}\left(\left(-2x+1\right)-1\right)$

$-1/2(-2x)$

$\boxed{x}$

equal! so $f(x)=g(x)$

Finding the Components of a Composition Function: Try to break up the function to find what part is the $f(x)$ and what part is the $g(x)$.

Example: Find the functions f and g such that

$f\circ g=H$ if $H(x)=\left(x^2+1\right)^{50}$

$f(x)=x^{50}$

$g(x)$ x^2+1

Example: Find the functions f and g such that

$f\circ g=H$ if $H(x)=\frac{3}{(x-5)^2}$

inside function will always include variable

$f\circ g=f(g(x))$

$f(x)=\frac{3}{x^2}$

$g(x)=x-5$