6.2 One-to-One Functions; Inverse Functions

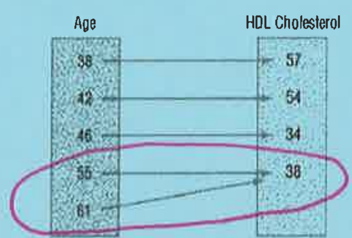
Determine if a Function is *One-to-One*:

- A function is one-to-one if for each y-value there is only 1 x-value that can be paired with it.

Example:

Determine whether the following functions are one-to-one.

(a) For the following function, the domain represents the age of five males and the range represents their HDL (good) cholesterol (mg/dL).



NOT one-to-one since 55 & 61 share 38

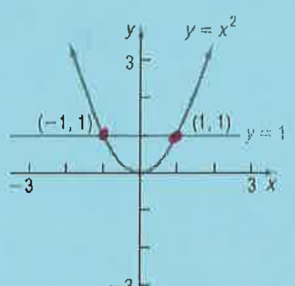
(b) $\{(-2, 6), (-1, 3), (0, 2), (1, 5), (2, 8)\}$

yes one-to-one

- Horizontal Line Test: if every horizontal line intersects the function f in at most 1 point. Then f is one-to-one

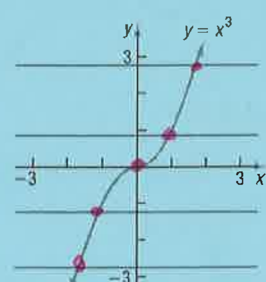
Example:

(a) $f(x) = x^2$



Not one-to-one

(b) $g(x) = x^3$



yes one-to-one

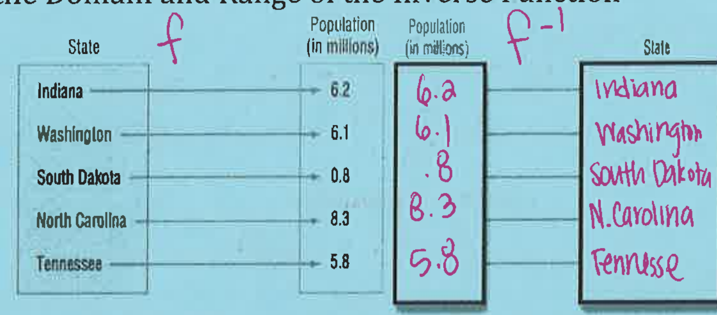
A function that is increasing on an interval I is a one-to-one function in I.
A function that is decreasing on an interval I is a one-to-one function on I.

Can you Explain WHY? Since the function isn't constant, or changing from increasing to decreasing; Then the function would always pass the horizontal line test

Determine the *Inverse* of a Function by Data:

- If a function is One-to-One then there exists an ~~a~~ Inverse function

Example: Find the inverse of the following. State the Domain and Range of the Inverse Function



The Domain of the Inverse is{6.2, 6.1, .8, 8.3, 5.8 }

Range of the Inverse: {Indiana, washington, S. Dakota, N. Carolina, Tenesse }

Example: Find the inverse of the following function. State the domain and range of the function and it's inverse: {(-5,1), (3,3), (0,0), (2,-4), (7, -8)}

f domain: { -5, 3, 0, 2, 7 }

f range: { 1, 3, 0, -4, -8 }

f *inverse* domain: { }

f *inverse* range: { }

Verifying Inverse Functions

$f^{-1}(f(x)) = x$ where x is in the domain of f
$f(f^{-1}(x)) = x$ where x is in the domain of f^{-1}

Example: Verify that the inverse of $g(x) = x^3$ is $g^{-1}(x) = \sqrt[3]{x}$

$g(g^{-1}(x)) = (\sqrt[3]{x})^3 = x$

$g^{-1}(g(x)) = \sqrt[3]{(x^3)} = x$

Since they both equal x they are inverse functions

Verifying Inverse Functions Continued:

Verify that the inverse of $f(x) = \dfrac{3}{x+5}$ is $f^{-1}(x) = \dfrac{3}{x} - 5$.

$$f^{-1}(f(x)) = \frac{3}{\left(\frac{3}{x+5}\right)} - 5 = \frac{\cancel{3}}{1} \cdot \frac{x+5}{\cancel{3}} - 5 = x+5-5 = \boxed{x}$$

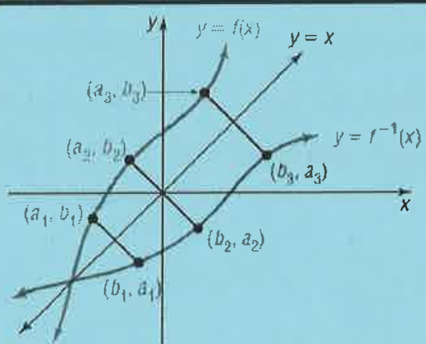
$$f(f^{-1}(x)) = \frac{3}{\left(\frac{3}{x}-5\right)+5} = \frac{3}{\frac{3}{x}} = \frac{\cancel{3}}{1} \cdot \frac{x}{\cancel{3}} = \boxed{x}$$

yes they are inverses

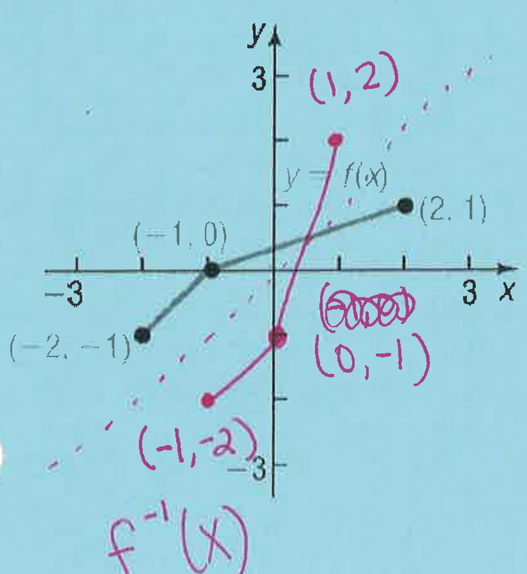
Determine the graph of the Inverse Function Based of the Graph of the Function:

The graph of a function f and the graph of its inverse f^{-1} are symmetric with respect to the line $y = x$.

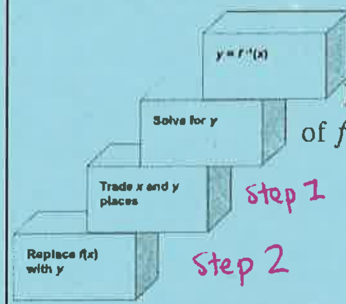
Example:



Example: Draw the graph of its inverse



Steps to Finding the Inverse of a Function:



Example: Find the inverse of $f(x) = 2x + 3$. Graph f and f^{-1}

step 1 $y = 2x+3$

Step 2 $x = 2y+3$

Step 3 $\quad -3 \qquad -3$

$\dfrac{x-3}{2} = \dfrac{2y}{2}$

$y = \dfrac{x-3}{2}$

step 4 $\boxed{f^{-1}(x) = \dfrac{x-3}{2}}$

Example: $f(x) = \dfrac{2x+1}{x-1}, \quad x \neq 1$

is one-to-one. Find its inverse and check the result.

① $y = \dfrac{2x+1}{x-1}$ ② $x = \dfrac{2y+1}{y-1}$

③ $(y-1)\, x = \dfrac{2y+1}{\cancel{(y-1)}}\cancel{(y-1)}$

$xy - x = 2y+1$

$-2y \quad +x \quad -2y \quad +x$

$xy - 2y = x+1$

$y(x-2) = x+1$

$y = \dfrac{x+1}{x-2}$

$\boxed{f^{-1}(x) = \dfrac{x+1}{x-2}}$

Summary:

1. If the function f is one to one then it has an inverse written as f^{-1}
2. The Domain of f = Range of f^{-1}.
   The Range of f = Domain of f^{-1}.
3. To verify that f^{-1} is an inverse of f, show that $f^{-1}(f(x)) = x$ for every x in the domain of f and $f(f^{-1}(x)) = x$ for every x in the domain of f^{-1}
4. The graph of f and f^{-1} are symmetric with respect to the line $y = x$
   identity function