6.3 Exponential Functions

Solving Exponential Equations:

Laws of Exponents

If s, t, a, and b are real numbers with $a > 0$ and $b > 0$, then

$$a^s \cdot a^t = a^{s+t} \qquad (a^s)^t = a^{st} \qquad (ab)^s = a^s \cdot b^s$$

$$1^s = 1 \qquad a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s \qquad a^0 = 1$$

Evaluate Exponential Functions:

1. The value of f doubles with every 1-unit increase in x
2. The value of f at $x = 0$, is 5 $f(0) = 5$

x	Work	f(x)	
0	$(5)2^0 =$	5	×2
1	$(5)2^1 =$	10	×2
2	$(5)2^2 =$	20	×2
3	$5(2)^3 =$	40	×2
4	$5(2)^4 =$	80	

so 2 is our multiplier & it's exponential

$y = 5(2)^x$

since were multiplying its exponential

Exponential Functions can be written as:

$$f(x) = Ca^x$$

- C represents the initial value (when x=0)
- a represents the multiplier

Where $a > 0$, $a \neq 1$, and $C \neq 0$. The Domain is all real #'s.

Ratio of Consecutive Outputs

$$\frac{f(x+1)}{f(x)} = a \quad \text{or} \quad f(x+1) = af(x)$$

Linear, Exponential, or Neither?

Average rate of change — Ratio of consecutive outputs

(a)

x	y	
−1	5	−3
0	2	−3
1	−1	−3
2	−4	−3
3	−7	

$\frac{\Delta y}{\Delta x} = \frac{2-5}{0-(-1)} = -3$

linear

$y = -3x + 2$

since we are adding its linear

not linear — $\frac{(x+1)}{x}$

(b)

x	y	
−1	32	×½
0	16	×½
1	8	×½
2	4	×½
3	2	

$\frac{16}{32} = \frac{1}{2}$ $\frac{8}{16} = \frac{1}{2}$ $\frac{8}{4}\left(\frac{1}{2}\right)$ $\frac{4}{2} = \frac{1}{2}$

exponential

$y = 16\left(\frac{1}{2}\right)^x$

since we are multiplying by $\frac{1}{2}$ it exponential

(c)

x	y	
−1	2	+2
0	4	+3
1	7	+4
2	11	+5
3	16	

NOT linear

$\frac{4}{2} = 2$ $\frac{7}{4} = \frac{3}{2}$

Neither

not exponential

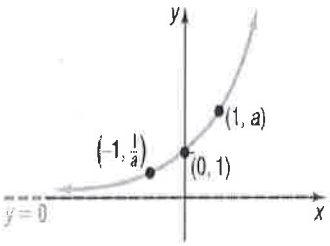
Graphing Exponential Functions:

Properties of the **Parent Exponential Function**

$$f(x) = a^x$$

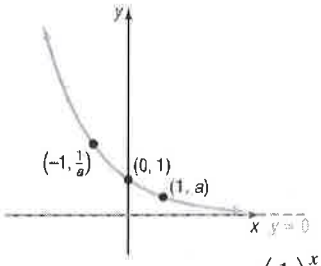
1. The **domain** is the set of all real numbers; **Range** is the set of all positive real numbers
2. There are no **x-intercepts**; the **y-intercept** is 1.
3. The x-axis (y=0) is **a horizontal asymptote**
4. $f(x) = a^x$
   a. $a > 1$, is **increasing** and is one-to-one.
   b. $0 < a < 1$, is **decreasing** and is one-to-one.
5. The graph of f contains the **points**: $(0,1)$, $(1,a)$ and $\left(-1, \frac{1}{a}\right)$
6. The graph of f is smooth and continuous with no corners or gaps

$f(x) = a^x, a > 1$



Example: $f(x) = 2^x$

$f(x) = a^x, 0 < a < 1$

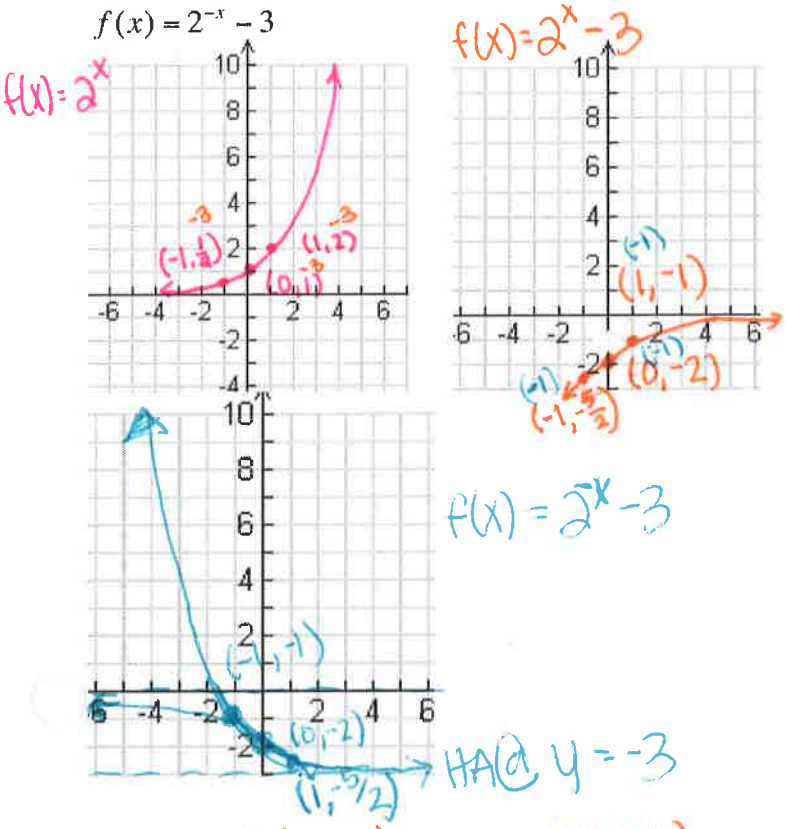


Example: $f(x) = \left(\frac{1}{2}\right)^x$

Graphing Exponential Functions using transformations:

All of the transformations that you learned apply to all functions, so what would the following look like?

$f(x) = 2^{-x} - 3$

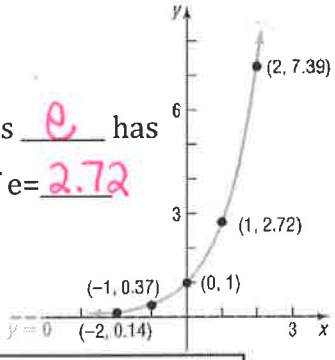


Domain: (-∞,∞) Range: (-2,∞)

Horizontal asymptote: y=-3

Define the number *e*:

The natural base written as e has the mathematical value of e=2.72



The **number** e is defined as the number that the expression

$$\left(1 + \frac{1}{n}\right)^n$$

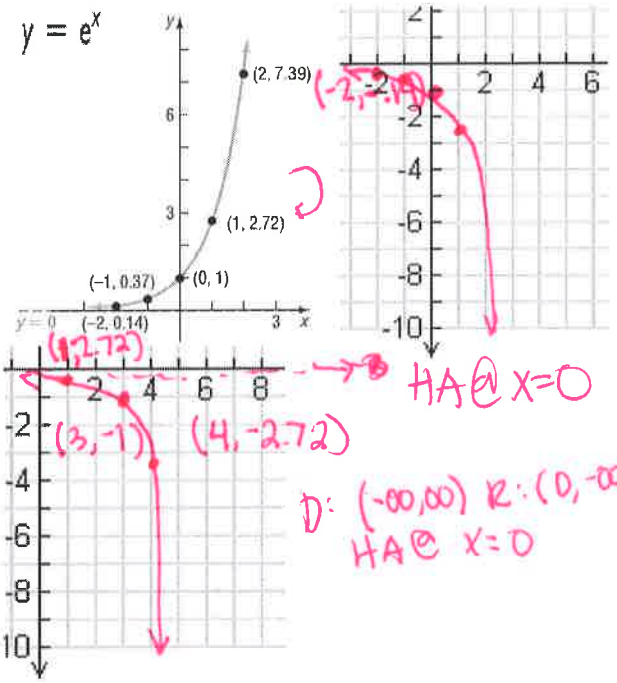
approaches as $n \to \infty$.

In calculus, this is expressed using limit notation as

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

Graph $f(x) = -e^{x-3}$ Using Transformations;

state the domain, range, and horizontal asymptote.



Solving Exponential Equations:

If $a^u = a^v$, then $u = v$

Solve each exponential equation.

(a) $2^{3x-1} = 32$

$2^{3x-1} = 2^5$

$3x-1=5$

$3x=6$

$x=2$

(b) $e^{2x-1} = \frac{1}{e^{3x}} \cdot \left(e^{-x}\right)^4$

$\frac{1}{e^{3x}} \cdot e^{-4x}$

$e^{-3x} \cdot e^{-4x} = e^{-7x}$

$e^{2x-1} = e^{-7x}$

$2x-1=-7x$

$9x=1$

$x=\frac{1}{9}$

Story Problem $f(t) = 1 - e^{-.02t}$

(a) What is the probability that a car will arrive within 5 min of 9:00 pm?

(b) Determine the probability the car will arrive within 30 min of 9:00 pm?

(c) Graph f using your calculator

(d) What value does f approach as t becomes unbounded in the positive direction?

a) $1 - e^{-.2(5)} \approx .6321 \approx 63.21\%$

b) $1 - e^{-.2(30)} \approx .9975 \approx 99.75\%$

c) 

d) as t gets large f(t) approaches 1