6.4 Properties of logarithms

- $y = e^x$ (Natural Base) is the amount of continuous growth after a certain amount of time.
- $y = \ln x$ (Natural Log) is the amount of time needed to reach a certain level of continuous growth

$$\boxed{b^y = x \quad \text{and} \quad \log_b x = y}$$

For example, $4 = \log_3 81$ is equivalent to $81 = 3^4$.

Change the following from exponential form to logarithmic form:

(a) $1.2^3 = m$ $\quad \log_{1.2} m = 3$

(b) $e^b = 9$ $\quad \log_e 9 = b$

(c) $a^4 = 24$ $\quad \log_a 24 = 4$

Change the following from logarithmic form to exponential form:

(a) $\log_a 4 = 5$ $\quad a^5 = 4$

(b) $\log_e b = -3$ $\quad e^{-3} = b$

(c) $\log_3 5 = c$ $\quad 3^c = 5$

Find the *exact value* of:

(a) $\log_3 81$

$3^x = 81$

$3^4 = 81$

$\boxed{4}$

(b) $\log_2 \frac{1}{8}$

$2^x = \frac{1}{8}$

$2^x = 8^{-1}$

$2^x = 2^{3(-1)}$

$\boxed{-3}$

$y = \log_a x$ (defining equation: $x = a^y$)
Domain: $0 < x < \infty$ Range: $-\infty < y < \infty$

To find the domain of f set the inside =0 then check on number line:

(a) $f(x) = \log_3(x-2)$ $\quad x-2>0$

$x-2=0$

$x \neq 2$

Test(1) $1-2>0$ $\quad -1>0$ F

Test (3) $3-2>0$ $\quad 1>0$ T

x F T

1 2 3

$(2, \infty)$

(b) $F(x) = \log_2\left(\frac{x+3}{x-1}\right)$ set denominator =0

$x-1=0$ $\quad x \neq 1$

set inside =0

$\frac{x+3}{x-1} = 0$ $\quad x+3=0$ $\quad x \neq -3$

Test (-4) $\frac{-4+3}{-4-1} = \frac{-1}{-5} = \frac{1}{5} > 0$

T F T

-3 1

Test(0) $\frac{0+3}{0-1} = -3 > 0$ F

Test(2) $\frac{2+3}{2-1} = \frac{5}{1} = 5 > 0$ T

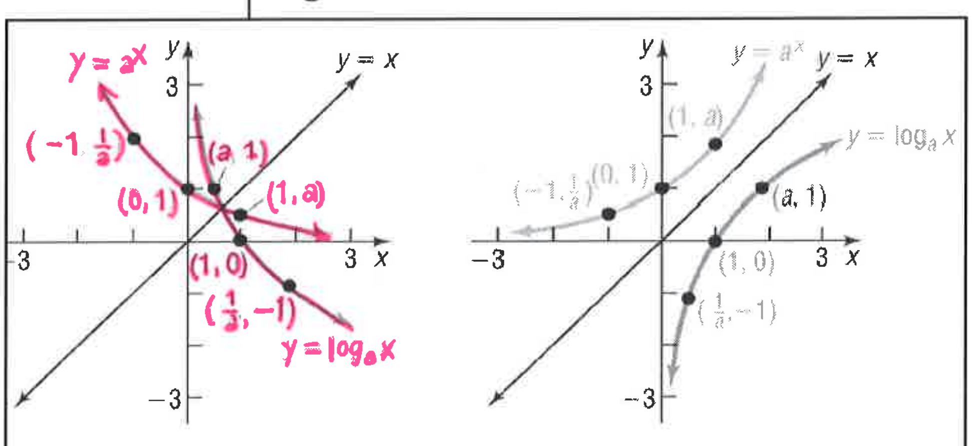
$(-\infty, -3) \cup (1, \infty)$

(c) $h(x) = \log_2|x-1|$

Since $|x-1|$ must be >0 & | | means that it turns everything positive then the only restriction is $x-1=0$ $\quad x \neq 1$

$(-\infty, 1) \cup (1, \infty)$

Logarithm Function *Inverses (switch each x and y)

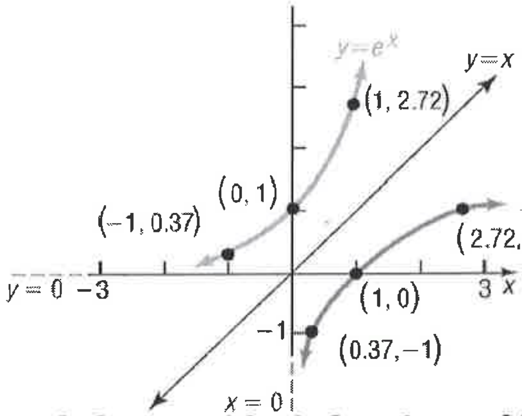


(a) $0 < a < 1$ $\quad$ (b) $a > 1$

remember $y = a^x$ & $y = \log_a x$ are inverse functions

Natural Logarithm Function

$$\boxed{y = \ln x \quad \text{and} \quad y = e^x}$$



Graph the Logarithmic function and its inverse:

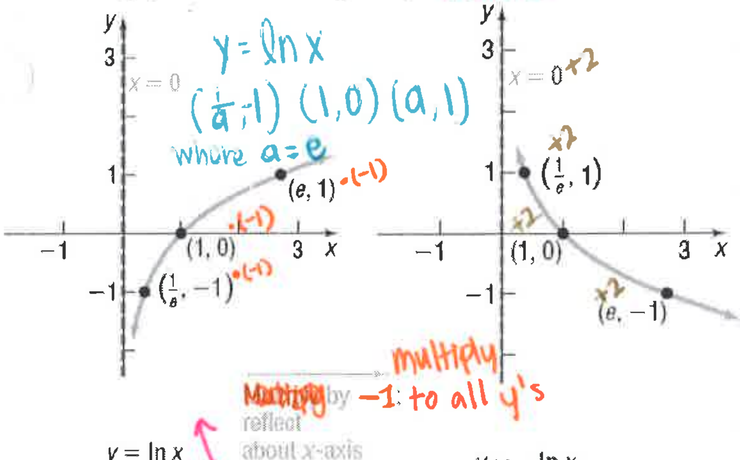
(a) Find the domain of $f(x) = -\ln(x-2)$ $(x-2) \neq 0$ $x \neq 2$ $(2, \infty)$

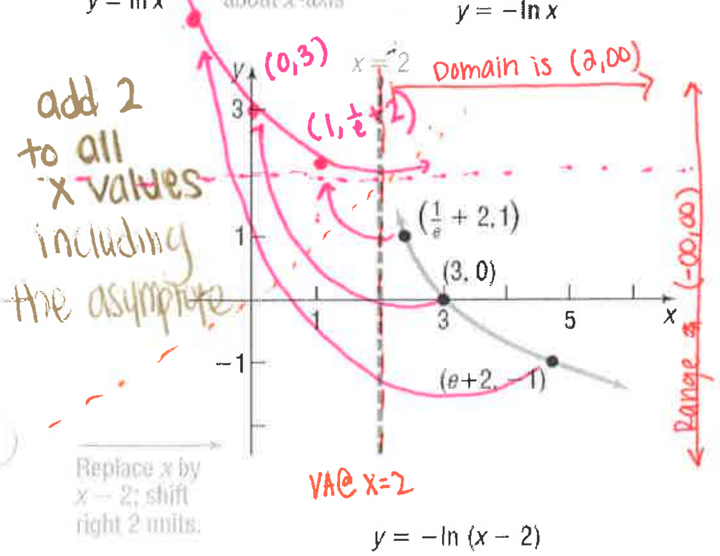
(b) Graph f

(c) Determine the range and vertical asymptote of f $(-\infty, \infty)$ VA@ $x=2$

(d) Find the points of f^{-1} and Graph

(e) Find the range of f $(2, \infty)$





Common Logarithm has a base of 10

$$\boxed{y = \log x \quad \text{if and only if} \quad x = 10^y}$$

Graph the Logarithmic function and its inverse:

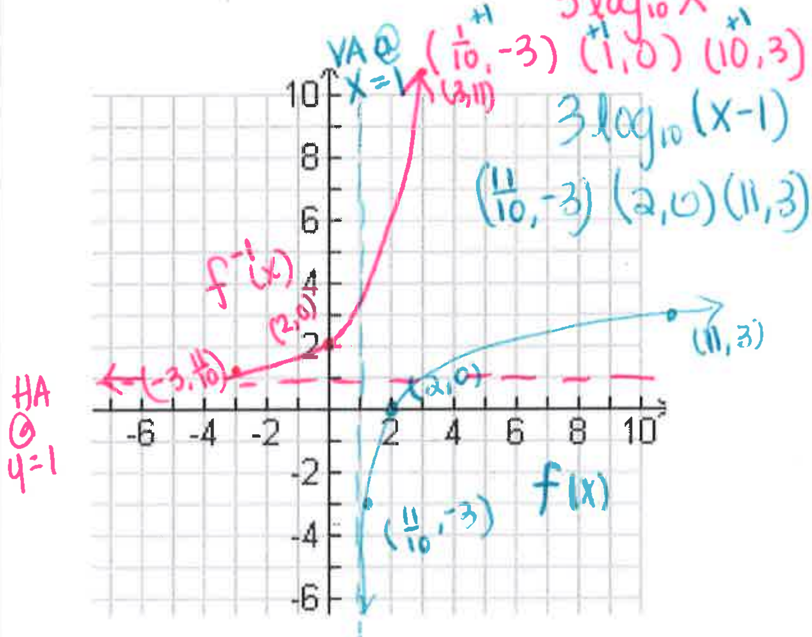
(a) Find the domain of $f(x) = 3\log(x-1)$ $x-1 \neq 0$ $x \neq 1$ $x > 1$ $(1, \infty)$

(b) Graph f

(c) Determine the range and vertical asymptote of f VA@ $x=1$ $(-\infty, \infty)$

(d) Find the points of f^{-1} and Graph $\log_{10} x$

(e) Find the range of f $(\frac{1}{10}, -1)$ $(1, 0)$ $(10, 1)$



$(a)\ \log_2(2x+1) = 3 \qquad (b)\ \log_x 343 = 3$

Solving Logarithmic Equations:

a) $2^3 = 2x+1$

$8 = 2x+1$

$7 = 2x$

$\boxed{x = \frac{7}{2}}$

b) $x^3 = 343$

$\sqrt[3]{x^3} = \sqrt[3]{343}$

$\boxed{x = 7}$

Using Logarithms to solve an Exponential Equation:

Solve: $\frac{2e^{3x}}{2} = \frac{6}{2}$ $e^{3x} = 3$

$\log_e 3 = 3x$

$\ln 3 = 3x$

exact value $\boxed{x = \frac{\ln 3}{3}}$

approximate $\boxed{x \approx .366}$