6.5 Properties of Logarithms

- ✓ Calculator logs are base 10.
- ✓ The natural log (ln) is base e.
- ✓ Exponential graphs increase at a faster and faster rate
- ✓ Logarithmic graphs increase at a slower and slower rate
  - ***Properties of Logarithms:***
  - ***Remember*** $\log_e x = \ln x$

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $b^{\log_b M} = M$
4. $\log_b b^r = r$
5. $e^{x \ln b} = b^x$
6. $\log_b (MN) = \log_b M + \log_b N$
7. $\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$
8. $\log_b M^r = r \log_b M$
9. *If* $M = N$, then $\log_b M = \log_b N$
10. *If* $\log_b M = \log_b N$, then $M = N$
11. *Change of base formulas*: $\log_b M = \frac{\log_t M}{\log_t b} = \frac{\log M}{\log b} = \frac{\ln M}{\ln b}$

Show that:

a. $\log_a 1 = 0$ change to exponential

$a^0 = 1$ since anything (except zero) raised to the zero power = 1

b. $\log_a a = 1$

Change to exponential

$a^1 = a$ since anything raised to the first power is itself

Using Properties

a. $\log_\pi \pi^3 =$ property #4 $\log_b b^r = r$

$\log_\pi \pi^3 = 3$

b. $5^{\log_5 \sqrt{3}}$ property #3 $b^{\log_b M} = M$

$5^{\log_5 \sqrt{3}} = \sqrt{3}$

c. $\ln e^{0.35t}$ $\log_e e^{.35t}$ property #4

$= .35t$

Write $\log_2\left(x^2\sqrt[3]{x-1}\right)$, $x>1$,
as a sum of logarithms.
Express all powers as factors.

$\log_2\left(x^2\sqrt[3]{x-1}\right)$

$\log_2 x^2 + \log_2 \sqrt[3]{x-1}$ property #6

$\boxed{2\log_2 x + \frac{1}{3}\log_2(x-1)}$ property #8

Write $\log_6 \dfrac{x^4}{\left(x^2+3\right)^2}$, $x \neq 0$,
as a difference of logarithms.
Express all powers as factors.

$\log_6 x^4 - \log_6 (x^2+3)^2$ property #7

$\boxed{4\log_6 x - 2\log_6(x^2+3)}$ property #8

Write $\ln \dfrac{x^3\sqrt{x-2}}{(x+1)^2}$, $x>2$,
as a sum and difference of logarithms.
Express all powers as factors.

$\ln x^3 + \ln\sqrt{x-2} - \ln(x+1)^2$ #6 & #7

$\boxed{3\ln x + \frac{1}{2}\ln(x-2) - 2\ln(x+1)}$ #8

Write each of the following as a single logarithm.

(a) $3\ln 2 + \ln\left(x^2\right)$ $\quad \ln 2^3 + \ln(x^2)$ $\quad \boxed{\ln 8x^2}$

(b) $\frac{1}{2}\log_a 4 - 2\log_a 5$ $\quad \log_a 4^{1/2} - \log_a 5^2 = \boxed{\log_a \frac{2}{25}}$

(c) $-2\log_a 3 + 3\log_a 2 - \log_a\left(x^2+1\right)$

$\log_a 3^{-2} + \log_a 2^3 - \log_a(x^2+1)$

$\log_a \frac{1}{9} + \log_a 8 - \log_a(x^2+1)$

$\boxed{\log_a \dfrac{8}{9(x^2+1)}}$

Approximate the following:
round answer to four decimal places.

a. $\log_3 12$. $\quad \dfrac{\log 12}{\log 3} \approx 2.789$

b. $\log_5 89$. $\quad \dfrac{\log 89}{\log 5} \approx 2.32$

c. $\log_{\sqrt{2}} \sqrt{5}$. $\quad \dfrac{\log\sqrt{5}}{\log\sqrt{2}} = \dfrac{\cancel{\frac{1}{2}}\log 5}{\cancel{\frac{1}{2}}\log 2} = \dfrac{\log 5}{\log 2}$

$\dfrac{\log 5}{\log 2} \approx 2.32$

Applying all Properties:

a. $\log_3 8 \cdot \log_8 9$

$\dfrac{\cancel{\log 8}}{\log 3}\cdot\dfrac{\log 9}{\cancel{\log 8}} = \dfrac{\log 9}{\log 3} = 2$

b. $e^{\log_{e^2} 81}$ $\quad e^u$

let $u = \log_{e^2} 81$

$= (e^2)^u = 81 \quad ((e^2)^u)^{1/2} = 81^{1/2}$

$e^u = 9 \quad \ln e^u = \ln 9 \Rightarrow \underline{u = \ln 9}$ plug back in

$e^{\ln 9} = \boxed{9}$