6.7 Financial Modeling

MEMORIZE THESE THEOREMS

Simple Interest Formula Just compounded once lump

$$I = Prt$$

I = Interest, P = principal, r = rate, t = time

Compound Interest Formula

↳ means interest is earning interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = Total Amount, P = principal, r = rate, n = # of time compounded per year, t = time

Continuous Compound

$$A = Pe^{rt}$$

A = Total Amount, P = Principal, r = rate, t = time

↳ mean continuous growth $e = 2.718281828\ldots$

Annually: $n=1$ once per year

Semi Annual: $n=2$ twice per/year

Quarterly: $n=4$ four times per year

Monthly Daily: $n=365$ 365 times per year

EX: A credit union pays interest of 4% per annum compounded quarterly on a certain savings plan. If $2000 is deposited in such a plan and the interest is left to accumulate, how much is in the account after 1 year?

Since we only know Simple Interest @ this point

$I = Prt$ since its compounded each Quarter

First Quarter Q1 $I = \$2000(.04)(\frac{1}{4}) = 20$ so the interest is $\$20$

Now we have a new principal of $2000 + 20 = \$2020$

so apply the new P to Q2

Q2 $I = \$2020(.04)(\frac{1}{4}) = \$20.20$ earned $\$20.20$ (interest earned interest)

New Principal $2020 + 20.20 = \$2040.20$

Q3 $I = \$2040.20(.04)(\frac{1}{4}) = \$20.40$

New principal $2040.20 + 20.40 = \$2060.60$

Q4 $I = \$2060.60(.04)(\frac{1}{4}) = \$20.61$

New Principal = $A = 2060.60 + 20.61 = \$2081.21$

Will you get the same answer if you use **Simple Interest formula** for each period and if you use the **Compound Interest Formula**? Yes

Can you see why you would want to use the Compound Interest Formula?

$A = P(1+\frac{r}{n})^{nt} = 2000(1+\frac{.04}{4})^{4(1)}$

$= \$2081.21$

EX: Which of the following yields the greatest value, if $1000 is invested at an annual rate of 10% compounded by the following periods:

$A = 1000(1+\frac{.10}{n})^{n(1)}$

* Plug in n the the equation above

Annually: $n=1$ $\$1100$

Semiannually: $n=2$ $\$1102.50$

Quarterly: $n=4$ $\$1103.81$

Monthly: $n=12$ $\$1104.71$

Daily: $n=365$ $\$1105.16$ *

compounded daily yeids greatest revenue

EX: Find the amount A that results from investing a principal P of $2000 at an annual rate r of 8% compounded continuously for a time t of 1 year.

$A = 2000e^{.08(1)}$ * remember: continuously mean $A = Pe^{rt}$

$A = \$2166.57$

so the interest gained $I = A - P$

was $A - P = 2166.57 - 2000$

$I = 166.57$

Effective Rate of Return
OR Effective Rate of Interest

These two phrases mean the same thing.
It basically means: "What would the simple interest rate have to be to earn the same amount as with compound interest (in 1 year)

EX: Suppose that you have $1000 and a bank offers to pay you 3% annual interest on a savings account with interest compounded monthly. In one year:

$P=1000$ $A=P(1+\frac{r}{n})^{nt}$ $n=12$ $r=.03$ $t=1$

$A=1000(1+\frac{.03}{12})^{12(1)}$

$=1030.42$

so the interest earned is $30.42. use

$I=Prt$ with $t=1$ $I=30.42$

& $P=\$1000$ to find the annual simple interest rate needed to earn the same revenue $30.42=1000r(1)=.03042$

so the effective rate of interest is 3.042%

EX: Suppose you want to open a money market account. You visit three banks to determine their money market rates. Bank A offers you 4% annual interest compounded daily, Bank B offers you 4.1% compounded monthly, and Bank C offers 3.95% compounded continuously. Determine which bank is offering the best deal.

you can pick any #

since there is no amount of Principal let $P=1$ or time so $t=1$

Bank A

$r=.04$
$n=365$
$A=P(1+\frac{r}{n})^{nt}$
$A=1(1+\frac{.04}{365})^{365(1)}$
$A=1.0408$

Bank B

$r=.041$
$n=12$
$A=1(1+\frac{.041}{12})^{12(1)}$
$A=1.04178$

Bank C

$r=.0395$
~~$n=$~~ continuous
$A=Pe^{rt}$
$A=1e^{.0395(1)}$
$A=1.04029$

* Bank B is offering you the best deal

EX: What annual rate of interest compounded quarterly should you seek if you want to double your investment in 6 years?

we want to find the rate r it takes to double our many in 6 yrs so

$A=P(1+\frac{r}{n})^{nt}$

since we don't have a specific # for P or A we can pick one, since we are doubling so

if $P=1$ then $A=2$
if $P=100$ then $A=200$

$r=?$ $n=4$ $t=6$

$2=1(1+\frac{r}{n})^{nt}$
$2=1(1+\frac{r}{4})^{4(6)}$
$2^{(\frac{1}{24})}=(1+\frac{r}{4})^{24(\frac{1}{24})}$

$2^{\frac{1}{24}}=(1+\frac{r}{4})$
$2^{\frac{1}{24}}-1=\frac{r}{4}$
$r=4(2^{\frac{1}{24}}-1)$
$r\approx .1171089$
$r=11.7\%$

EX:

(a) How long will it take for an investment to double in value if it earns 6% compounded continuously?

$A=Pe^{rt}$ Find $t=?$

$2=1e^{.06(t)}$ $\Rightarrow$ $2=e^{.06t}$ $\Rightarrow$ $\ln 2=\ln e^{.06t}$

$\ln 2=.06t \Rightarrow \frac{\ln 2}{.06}=t \approx 11.55$ yrs

(b) How long will it take to triple at this rate?

since we are tripling have $P=1$ & $A=3$

or $P=100$ & $A=300$ ect

$3=1e^{.06t}$
$\ln 3=\ln e^{.06t}$
$\ln 3=.06t$

$t=\frac{\ln 3}{.06}\approx 18.31$ years