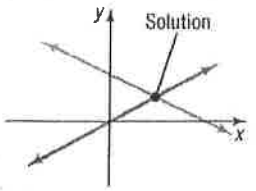
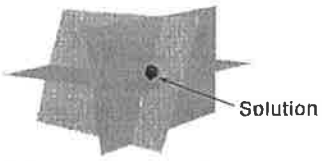
8.1 Systems of Linear Equations; Substitution and Elimination

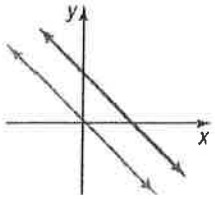


(a) Intersecting lines; system has one solution

If the lines intersect then there is 1 solution given at the point of intersection. The system of equations is Consistent and the equations are independent.

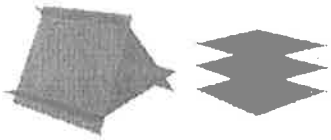


(a) Consistent system; one solution

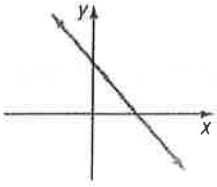


(b) Parallel lines; system has no solution

If the lines are parallel then it has NO solution because the points never intersect. The system of equations is inconsistent

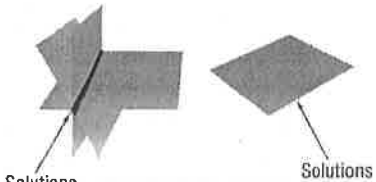


(c) Inconsistent system; no solution



(c) Coincident lines; system has infinitely many solutions

If the lines are coincident then it has infinitely many solutions because they intersect at every point. The system of equations is Consistent and the equations are dependent



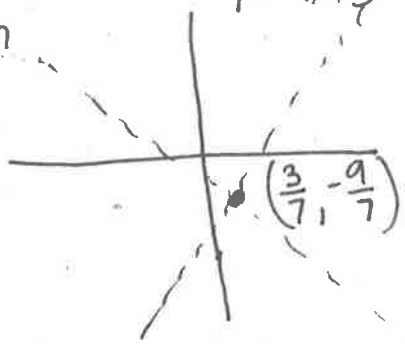
(b) Consistent system; infinite number of solutions

Solve: $\begin{cases} 2x+3y=-3 \\ 4x-y=3 \end{cases}$

$y=-\frac{2}{3}x-1$

$y=4x+3$

graph



Substitution

$2x+3(4x+3)=-3$

$2x+12x+9=-3$

$\frac{14x}{14}=\frac{-12}{14}$ $\quad x=-\frac{9}{7}$

$4\left(\frac{-9}{7}\right)-y=3$ $\quad y=\frac{3}{7}$

$\frac{21}{7}\cdot\frac{3}{1}-\frac{36}{7}-y=3=y$

Elimination

A Movie Theater sells tickets for $8.00 with seniors receiving a discount of $2. One evening the theater sold 525 tickets and took in $3580 in revenue. How many of each type of ticket were sold?

Need 2 equations

$x+y=525 \rightarrow y=525-x$

$8x+6y=3580$

$8x+6(525-x)=3581$

$8x+3150-6x=3581$

$2x=430$ $\quad x=215$

$215+y=525$

$y=310$

They sold 215 regular tickets & 310 senior tickets

Solve: $\begin{cases} 2x+y=5 \\ 4x+2y=8 \end{cases}$ $-2(2x+y=5) \rightarrow -4x-2y=-10$

$4x+2y=8$

$0+0=-2$

False

since $0 \neq -2$

Inconsistent

Solve: $\begin{cases} 2x+y-z=-2 \\ x+2y-z=-9 \\ x-4y+z=1 \end{cases}$

$2x+y-z=-2$

$x-4y+z=1$

$3x-3y=-1$ ★

$2x+y-z=-2$

$-x-2y+z=9$ (mult by -1)

$x-y=7$ ★

$3x-3y=-1$

$-3x+3y=-21$ (mult by -3)

$0=-28$

False

Inconsistent

Solve: $\begin{cases} 2x+y=4 \\ -6x-3y=-12 \end{cases}$ $3(2x+y=4) \rightarrow 6x+3y=12$

$-6x-3y=-12$

$0=0$

True

Consistent, Dependent

so solve an equation for x

$2x+y=4$

$2x=-y+4$

$x=-\frac{1}{2}y+2$

so $\{(x,y) \mid x=-\frac{1}{2}y+2, y \in \mathbb{R}\}$

Find real numbers a, b, c so that the graph of the quadratic function $y = ax^2 + bx + c$ contains the points (-1,-4), (1,6), and (3,0).

Plug each point one at a time into to get all 3 equations

$-4=a(-1)^2+b(-1)+c$

$-4=a-b+c$ ★

$6=a(1)^2+b(1)+c$

$6=a+b+c$ ★

$3=a(0)^2+b(0)+c$

$3=c$ ★ since $c=3$ plug it into other 2 equations

$-7=a-b$

$3=a+b$

now we have 2 equations & 2 variables

$-4=2a$ $a=2$

$3=(-2)+b$ $b=5$

so equation is $y=-2x^2+5x+3$

Solve: $\begin{cases} 1. & x+y-z=-1 \\ 2. & 4x-3y+2z=16 \\ 3. & 2x-2y-3z=5 \end{cases}$

★ main idea: turn it into 2 equations w/ 2 variables

- equations 1 & 2

get rid of a variable to get an equation w/ only 2 variables

$x+y-z=-1$ multiply 1st eq. by -4

$4x-3y+2z=16$

$-4x-4y+4z=4$

$4x-3y+2z=16$

$-7y+6z=20$ ★ first equation w/ 2 variables

- equations 1 & 3

get rid of the same variable to get the 2nd equation w/ only 2 variables

$x+y-z=-1$ multiply 1st eq by -2

$2x-2y-3z=5$

$-2x-2y+2z=2$

$2x-2y-3z=5$

$-4y-z=7$ ★ 2nd equation w/ 2 variables

$-7y+6z=20$

$-4y-z=7$ ← times by $+6$

$-7y+6z=20$

$-24y-6z=42$

$-31y=62$

$y=-2$

plug into one of the 2 new equations

$-4(-2)-z=7$

$+8-z=7$

$z=1$

$x-2-1=-1$ $x=2$

$(2,-2,1)$ solution

Solve

$x - 2y - z = 8$ → mult by -2 → $-2x + 4y + 2z = -16$

$2x - 3y + z = 23$ → $2x - 3y + z = 23$

$4x - 5y + 5z = 53$

$y + 3z = 7$ *

mult by -4

$-4x + 8y + 4z = -32$

$4x - 5y + 5z = 53$

$3y + 9z = 21$ This can be simplified to

$\frac{3y}{3} + \frac{9z}{3} = \frac{21}{3}$

$y + 3z = 7$ * ↔ notice they are the same

Thus Dependent so

now write solutions in set builder notation

by solving for each variable

Since

first to find x pick any equation & substitute the new equation in for one of the two variables left

first equation $x - 2y - z = 8$

now take the new equation $y + 3z = 7$ & solve for y

$y = -3z + 7$

Then plug it in for y

$x - 2(-3z + 7) - z = 8$

$x + 6z - 14 - z = 8$

$x + 5z = 22$

$x = -5z + 22$

Thus

$$\{(x, y, z) \mid x = -5z + 22, y = -3z + 7, z \in \mathbb{R}\}$$

A doctor's prescription calls for the creation of pills that contain 12 units of Vitamin B12 and 12 units of Vitamin E. Your pharmacy stocks two powders that can be used to make these pills: #1 contains 20% B12 and 30% E, #2 contains 40% B12 and 20% E. How many units of each powder should be mixed in each pill? two variables so we need two equations let x = units of B12, y = units of E

$.2x + .4y = 12$
$.3x + .2y = 12$ > lets mult by 10 so we can get rid of decimals

$2x + 4y = 120$
$3x + 2y = 120$ → mult by -2

$2x + 4y = 120$
$-6x - 4y = -240$

$-4x = -120$
$x = 30$

Now plug $x=30$ into one of the equations

$2(30) + 4y = 120$
$60 + 4y = 120$
$4y = 60$
$y = 15$

Should use 30 units of powder 1 and 15 units of Powder 2

A B C

Three painters, Beth, Bill, and Eddie, working together can paint the exterior of a home in 10 hours. Bill and Eddie together have painted a similar house in 15 hours. One day all three worked on the same kind of house for 4 hours, then Eddie left. Beth and Bill took 8 more hours to finish. Assuming no gain or loss in efficiency, how long should it take each person to complete the job alone?

$\frac{1}{A} + \frac{1}{B} + \frac{1}{C} = \frac{1}{10}$
$\frac{1}{B} + \frac{1}{C} = \frac{1}{15}$
$4\left(\frac{1}{A} + \frac{1}{B} + \frac{1}{C}\right) + 8\left(\frac{1}{A} + \frac{1}{B}\right) = 1$ } 3 variable w/ 3 equations

Use 1st & 2nd equations - substitution
since $\frac{1}{B} + \frac{1}{C} = \frac{1}{15}$ then $\frac{1}{A} + \left(\frac{1}{15}\right) = \frac{1}{10}$
$\frac{1}{A} = \frac{1}{30}$ so $A = 30$

Now use equation 1 & 3 since $\frac{1}{A} + \frac{1}{B} + \frac{1}{C} = \frac{1}{10}$ plug into equation 3
$4\left(\frac{1}{10}\right) + 8\left(\frac{1}{A} + \frac{1}{B}\right) = 1$ since $A = 30$
$\frac{4}{10} + 8\left(\frac{1}{30} + \frac{1}{B}\right) = 1$
$8\left(\frac{1}{30} + \frac{1}{B}\right) = \frac{6}{10}$
$\frac{1}{30} + \frac{1}{B} = \frac{3}{40}$
$\frac{1}{B} = \frac{1}{24}$ $B = 24$

* Now plug into original equation
$\frac{1}{24} + \frac{1}{C} = \frac{1}{15}$
$\frac{1}{C} = \frac{1}{40}$ $C = 40$

So it would take Beth 30 hrs Bill 24 hrs & Eddie 40 hrs

Carletta has $10,000 to invest. As her financial consultant, you recommend that she invest in Treasury Bills that yield 6%, Treasury Bonds that yield 7%, and Corporate Bonds that yield 8%. Carletta wants to have $680 annually from her investments. The amount invested in Corporate Bonds must be half of that invested in Treasury Bills. Find the amount in each investment.

x = Treasury Bills $
y = Treasury Bonds $
z = Corporate Bonds $

$x + y + z = 10000$
$.06x + .07y + .08z = 680$ } 3 equations 3 variables
$x = 2z$

Substitute the 3rd equation into the 1st eq.
$2z + y + z = 10{,}000$
$3z + y = 10{,}000$ *

Substitute 3rd eq into 2nd eq
$.06(2z) + .07y + .08z = 680$
$.12z + .07y + .08z = 680$
$.2z + .07y = 680$
$20z + 7y = 68000$ *

$20z + 7y = 68000$
$-21z - 7y = -70000$ ← multiply by -7
$-z = -2000$
$z = 2000$

$x = 2(2000)$
$x = 4000$

$10000 - 2000 - 4000$
$y = 4000$

Must invest
$4000 in Treasury Bills
$4000 in Treasury Bonds
& $2000 in Corporate Bonds