8.3 Cramer's Rule: Determinants

Determinants only work on ______________ matrices.
ex: 2 x 2, 3 x 3, 4 x 4, …

Given a 2 x 2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

The Determinant: $D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Evaluate: $\begin{vmatrix} -2 & 3 \\ 4 & -1 \end{vmatrix}$

Cramer's Rule : For $\begin{cases} ax + by = s \\ cx + dy = t \end{cases}$ when

$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, $D_x = \begin{vmatrix} s & b \\ t & d \end{vmatrix}$ and $D_y = \begin{vmatrix} a & s \\ c & t \end{vmatrix}$

if $D \neq 0$, then $x = \dfrac{D_x}{D}$, and $y = \dfrac{D_y}{D}$

Solving a Sys. Of Eq. Using Determinants:

Ex: $\begin{cases} 3x - 6y = 24 \\ 5x + 4y = 12 \end{cases}$

Evaluating 3 x 3 determinants

For an *n* x *n* determinant A, the **cofactor** of entry a_{ij},

denoted by A_{ij}, is given by $A_{ij} = (-1)^{i+j} M_{ij}$.

Where is M_{ij} *the minor of entry* a_{ij}.

The 2 x 2 ____________ are called ________ of the 3 x 3 determinant

It **doesn't** matter which column or row you pick you will get the same ______________

Lets pick column 2 for this example

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} +$$

$$(-1)^{2+2} a_{22} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{3+2} a_{32} \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$$

Finding the Minor of a 3 x 3 Determinant:

For the Determinant $A = \begin{vmatrix} -1 & 2 & 0 \\ 3 & 2 & 1 \\ -2 & 1 & -1 \end{vmatrix}$

Find a) M_{23}

Evaluating the determinant of a 3 x 3 matrix

Find the value of the 3 by 3 determinant: $\begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 1 \\ 2 & 6 & 7 \end{vmatrix}$

Cramer's Rule:
For a system of 3 equations with 3 variables

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = c_1 \\ a_{21}x + a_{22}y + a_{23}z = c_2 \\ a_{31}x + a_{32}y + a_{33}z = c_3 \end{cases}$$

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

Cramer's Rule for Three Equations Containing Three Variables

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad z = \frac{D_z}{D}$$

where

$$D_x = \begin{vmatrix} c_1 & a_{12} & a_{13} \\ c_2 & a_{22} & a_{23} \\ c_3 & a_{32} & a_{33} \end{vmatrix} \qquad D_y = \begin{vmatrix} a_{11} & c_1 & a_{13} \\ a_{21} & c_2 & a_{23} \\ a_{31} & c_3 & a_{33} \end{vmatrix} \qquad D_z = \begin{vmatrix} a_{11} & a_{12} & c_1 \\ a_{21} & a_{22} & c_2 \\ a_{31} & a_{32} & c_3 \end{vmatrix}$$

Cramer's Rule with Inconsistent or Dependent Systems

- If $D = 0$ and at least one of the determinants D_x, D_y, or D_z is different from 0, then the system is inconsistent and the solution set is $\emptyset$ or $\{\ \}$.
- If $D = 0$ and all the determinants D_x, D_y, and D_z equal 0, then the system is consistent and dependent so that there are infinitely many solutions. The system must be solved using row reduction techniques.

Use Cramer's Rule to solve the following:

$$\begin{cases} x + 2y + z = 1 \\ 3x + 5y + z = 3 \\ 2x + 6y + 7z = 1 \end{cases}$$