Name Key Final

Math 1050 — Concurrent Enrollment 2012-13— Final Exam

Circle your high school:

High School
Parowan High School(Warr)
Cedar High School(Whetman)
Canyon View High School(~~Green~~) HULET
~~Bryce Valley High School(Lefevre)~~
~~Panguitch High School(Lefevre)~~
~~Delta High School(Fowles)~~
~~Open High School(Van Ausdal)~~

Instructions:

- Books, notes, calculators with symbolic manipulation, phones, and computers of any type are **NOT** allowed.
- *To receive full credit you must show all necessary work and provide clear explanations.*
- If the statement of a problem uses words like 'find', 'solve', or 'determine' you are expected to give a clear explanation of the mathematical steps and reasoning involved in 'finding', 'solving', or 'determining'.
- Expressions such as $\frac{12}{3}$, $\sqrt{12}$, $\log_2(4)$, etc. must be simplified for full credit.
- Neatly write your solutions directly on the exam paper. If a solution requires more space than given, you may continue on the back of the page. Work on scratch paper will not be graded.
- Please write your name on the cover sheet and both portions of your exam.

For administrative use only:

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Total
Possible	15	5	9	5	18	6	6	6	21	5	6	9	6	4	6	4	8	6	145
Score																			

Name: Key

CE MATH 1050 - Final Exam -
NO CALCULATOR

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- *To receive full credit you must show all necessary work and provide clear explanations.*
- Books, notes, **calculators**, computers, cell phones, and other internet-enabled devices are not allowed.
- When you have completed this section, please raise your hand so the proctor will give you the calculator section of the exam.

1. Each of these problems is worth 3 points. NO justification is required for these problems.

3 (a) For the polynomial $p(x) = (x-3)^2(x+1)^3$, list each real zero and its multiplicity.

(+1) $x = 3$ multiplicity of 2 (+1)
(+1) $x = -1$ multiplicity of 3

3 (b) Find $\log_{16} 4$. use $\log_b b^x$ rule (+1) $16^x = 4$ since $4^2 = 16$ then $\sqrt{16} = 4$ Thus $16^{\frac{1}{2}} = 4$ so $16^{1/2} = 4$

(+1) $\log_{16} 16^{1/2} = \boxed{\frac{1}{2}}$ (+1)

3 (c) Determine whether the equation $4x^2 + 4x + 9y^2 = 9$ is an ellipse, hyperbola, parabola, or circle.

(+1) $4(x^2 + x) + 9(y)^2 = 9$

$4(x^2 + x + (\frac{1}{2})^2) + 9(y)^2 = 9 + 4(\frac{1}{2})^2$

$4(x + \frac{1}{2})^2 + 9(y)^2 = 9 + 1$

$\frac{2\cancel{4}(x+\frac{1}{2})^2}{5\cancel{10}} + \frac{9(y)^2}{10} = \frac{10}{10}$

(+1) $\frac{2(x+\frac{1}{2})^2}{5} + \frac{9(y)^2}{10} = 1$

$\boxed{\text{Ellipse}}$ (+1)

3 (d) For the matrices $A = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 3 & -1 \end{bmatrix}$, find $A + 2B$.

$A = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$ $2B = \begin{bmatrix} 0\cdot 2 & 1\cdot 2 \\ 3\cdot 2 & -1\cdot 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 6 & -2 \end{bmatrix}$ (+1) $A + 2B = \begin{bmatrix} -1+0 & -3+2 \\ 2+6 & 0+(-2) \end{bmatrix}$ (+1) $= \begin{bmatrix} -1 & -1 \\ 8 & -2 \end{bmatrix}$ (+1)

3 (e) The matrix $A = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$ has inverse matrix $A^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$. Use the inverse matrix to solve the system of equations $\begin{cases} 7x - 3y = 3 \\ -2x + y = 1 \end{cases}$.

$[A][X] = [B]$ so $[A]^{-1}[B] = [X]$ (times each side by A^{-1} to get X by itself)

$X = \begin{bmatrix} x \\ y \end{bmatrix}$ $A = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ solutions

so (+1) $X = [A]^{-1}[B]$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ (+1) $= \begin{bmatrix} 1\cdot 3 + 3\cdot 1 \\ 2\cdot 3 + 7\cdot 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 13 \end{bmatrix}$ so $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 13 \end{bmatrix}$ (+1) $\boxed{\begin{matrix} x = 6 \\ y = 13 \end{matrix}}$

5 2. Write the circle $x^2 - 12x + y^2 + 27 = 0$ in standard form by completing the square.

+1 +1 +1

+1 $x^2 - 12x + \left(\frac{-12}{2}\right)^2 + (y)^2 = -27 + \left(\frac{-12}{2}\right)^2$

+1 $(x-6)^2 + (y)^2 = 9$

$\frac{(x-6)^2}{9} + \frac{y^2}{9} = 1$

+1 $\boxed{\frac{(x-6)^2}{9} + \frac{y^2}{9} = 1}$

since these are the same its a circle

9 3. For $p(x) = x^3 + x^2 - 8x - 6$ do the following:

3 (a) Use synthetic division to show that -3 is a zero of $p(x)$.

+1
$$\begin{array}{r|rrrr} -3 & 1 & 1 & -8 & -6 \\ & \downarrow & -3 & 6 & 6 \\ \hline & 1 & -2 & -2 & \boxed{0} \end{array}$$

+1 +1

$x^2 - 2x - 2$

since the remainder is zero -3 is a zero of the polynomial $x^3 + x^2 - 8x - 6$

6 (b) Find all zeros of $p(x)$.

use the reduced equation $(x+3)(x^2 - 2x - 2)$ +1

factor

$(x + \quad)(x - \quad)$

can't factor so use quadratic formula

$a=1 \quad b=-2 \quad c=-2$

+1 $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$

$x = \frac{2 \pm \sqrt{12}}{2}$ → simplify $\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

$x = \frac{2 \pm 2\sqrt{3}}{2}$ simplify $\frac{\cancel{2}(1 \pm \sqrt{3})}{\cancel{2}}$ $x = 1 \pm \sqrt{3}$

so $x = -3$ from part (a)

$x = 1 + \sqrt{3}$

$x = 1 - \sqrt{3}$ +1 +1 +1

$\boxed{\{x \mid x = -3, 1-\sqrt{3}, 1+\sqrt{3}\}}$

5 4. Find $\sum_{k=1}^{25}(3k+5)$.

+1 $= \sum_{k=1}^{25} 3k + \sum_{k=1}^{25} 5$

+1 $= 3\sum_{k=1}^{5} k + \sum_{k=1}^{25} 5$

+1 +1 +1

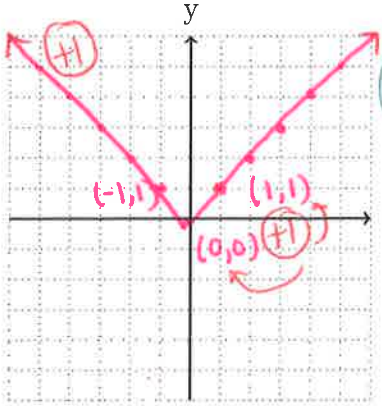
$= 3\left[\frac{5(5+1)}{2}\right] + 5(25) = 45 + 125 = \boxed{170}$

5. For each of the functions below, graph the basic function (for example $y = x^2$). Then graph each function. Label at least two points on each graph, and any asymptotes.

6 (a) $h(x) = -\left|\frac{1}{2}x\right|$

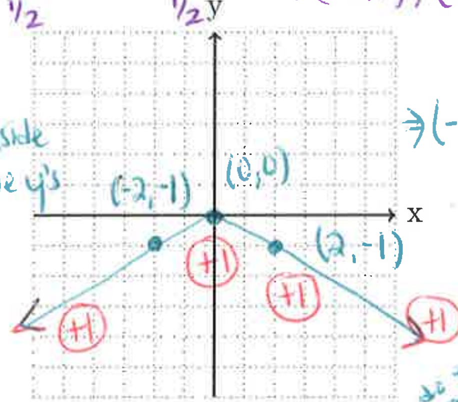
$\left|\frac{1}{2}x\right|$ inside parent function will effect the x values inside parent function does the opposite so instead of timesing by $\frac{1}{2}$ $\boxed{\text{divide by } \frac{1}{2}}$

$\left(\frac{-1}{1/2}, \frac{1}{}\right)$ $\left(\frac{0}{1/2}, 0\right)$ $\left(\frac{1}{1/2}, 1\right)$ → $(-2, 1^{(-1)}), (0, 0^{(-1)}) (2, 1^{(-1)})$



2 Basic Function = $|x|$

$-\left|\frac{1}{2}x\right|$ its on the outside so it effects the y's so mult all y's by -1



→ $(-2,-1), (0,0), (2,-1)$

4 $h(x) = -\left|\frac{1}{2}x\right|$

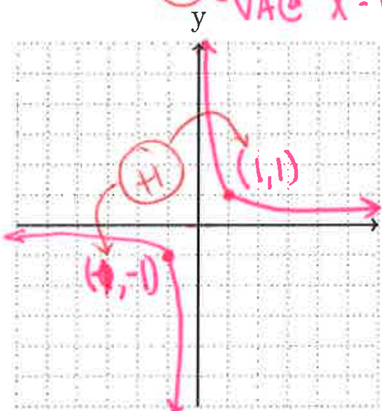
6 (b) $g(x) = \dfrac{1}{x-1} - 3$

HA @ $y=0$
VA @ $x=0$

$g(x) = \dfrac{1}{x-1}$ inside parent function so you do the opposite to all x's add 1 to all x's

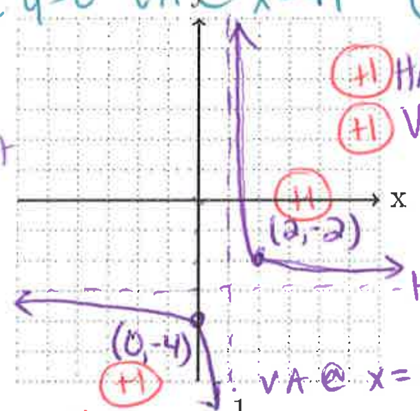
HA @ $y=0$ VA @ $x=0^{+1}$ $(-1^{+1}, -1)$, $(1^{+1}, 1)$

HA @ $y=0^{-3}$ VA @ $x=+1$ $(0, -1^{-3})$ $(2, 1^{-3})$



2 Basic Function = $\frac{1}{x}$

$g(x) = \dfrac{1}{x-1} - 3$ out side parent function so subtract 3 to all y values



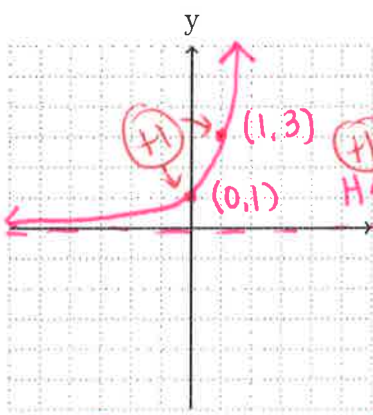
HA @ $y=-3$
VA @ $x=1$
$(0,-4)(2,-2)$

4 $g(x) = \dfrac{1}{x-1} - 3$

6 (c) $f(x) = 3^{-x} + 1$

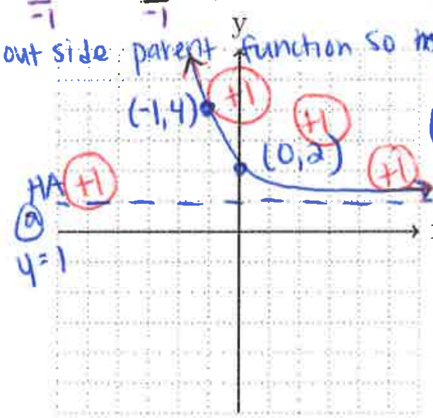
3^{-x} inside parent function so do opposite to all x's so we will divide (instead of mult) all x's by -1

HA @ $y=0$ $\left(\frac{0}{-1}, 1\right)$ $\left(\frac{1}{-1}, 3\right)$ ⇒ HA @ $y=0_{+1}$ $(0, 1^{+1}), (-1, 3^{+1})$



2 Basic Function = 3^x

$3^{-x} + 1$ out side parent function so add 1 to all y's

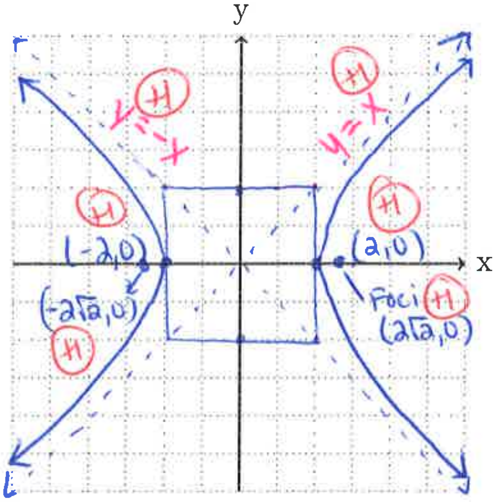


HA @ $y=1$
$(0,2)(-1,4)$

4 $f(x) = 3^{-x} + 1$

6. 6 Graph $\frac{x^2}{4} - \frac{y^2}{4} = 1$. Label the vertices, foci, and asymptotes. Since x^2 & y^2 & since we have a − sign its a $\boxed{\text{hyperbola}}$

since x^2 comes first it opens towards the x axis



$a^2=4$ so $a=2$
$b^2=4$ $b=2$

verticies at $(-2,0)$ & $(2,0)$

Foci @ $(-2\sqrt{2},0)$ & $(2\sqrt{2},0)$

& $y=\frac{2}{2}x = \boxed{y=x}$
$y=\frac{2}{-2}x = \boxed{y=-x}$ asymptotes

+1/2

$b^2 = c^2 - a^2$

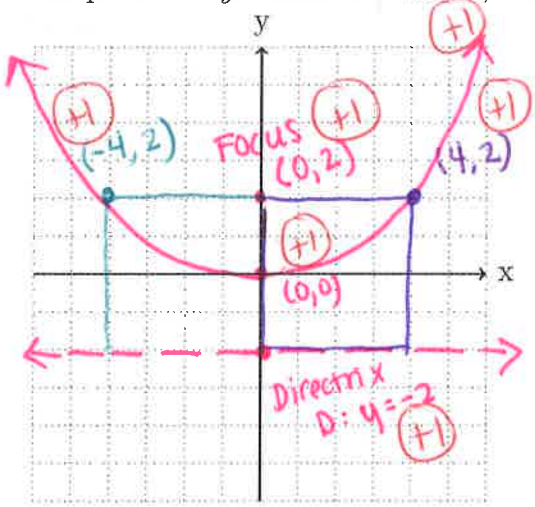
$4 = c^2 - 4$
$+4 \quad +4$

$c^2 = 8 \quad c=\sqrt{8}$

$\sqrt{4}\cdot\sqrt{2}$

$\boxed{c=\pm 2\sqrt{2}}$

6 7. Graph $x^2 = 8y$. Label the vertex, directrix, and focus.



$x^2 = 4ay$ so $a=2$

6 8. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 3 & -1 \end{bmatrix}$.

(+1) $\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 3 & -1 & 0 & 0 & 1 \end{array}\right]$ (+1) $R_2 = -R_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 3 & -1 & 0 & 0 & 1 \end{array}\right]$ ~~$R_3 = -3R_2 + R_3$~~

$R_3 = -3R_2 + R_3$ (+1) $\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 3 & 1 \end{array}\right]$ $R_3 = -R_3$ (+1) $\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -3 & -1 \end{array}\right]$

$R_1 = -R_3 + R_1$ (+1) $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -3 & -1 \end{array}\right]$ so $A^{-1} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & -1 & 0 \\ 0 & -3 & -1 \end{bmatrix}$ (+1)

Name:____________________________

CE MATH 1050 -

Calculator allowed

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2| 9. Each of these problems is worth 3 points. NO justification is required for these problems.

2 (a) Let $f(x) = 3x + 1$ and $g(x) = \sqrt{x-1}$. Find $(g - f)(x)$. $g(x) - f(x)$

$(g-f)(x) = (\sqrt{x-1}) - (3x+1)$ +1

$\boxed{= \sqrt{x-1} - 3x - 1}$ +1

3 (b) Let $f(x) = 3x + 1$ and $g(x) = \sqrt{x-1}$. Find $(f \circ g)(5)$.

$f(g(5))$

$g(5) = \sqrt{(5)-1} = \sqrt{4} = \boxed{2}$

+1 $f(g(5)) = f(2) = 3(2) + 1 = \boxed{7}$ +1 +1

2 (c) A degree 5 polynomial $p(x)$ with real coefficients has zeros 1, $2i$, and $3 + 5i$. Find the remaining zeros of p.

must have 5 solutions

$\boxed{x = 1, 2i, -2i, 3+5i, 3-5i}$ +1 +1

Their conjugates must also be solutions

4 (d) Find the determinant $\begin{vmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 3 \end{vmatrix}$

remember

+ − + −
− + − +
+ − + −
− + − +

Pick any row or column Pick ones w/ most zeros since theres 3 in a row I will have 3 minors

$0\begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} - 0\begin{vmatrix} 0 & 3 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix}$ +1

$0(6-6) - 0(0-3) + 1(0-2) = \boxed{-2} = D$

+1 0 − +1 0 − 2 +1 = −2

3 (e) Find a formula for the n^{th} term of the geometric sequence $18, -6, 2, -\frac{2}{3}, \ldots$

$\frac{-6}{18} = -\frac{1}{3}$ so $r = -\frac{1}{3}$ +1

$\frac{2}{-6} = -\frac{1}{3}$ $a = 18$ +1

$\boxed{a_n = 18\left(-\frac{1}{3}\right)^{n-1}}$ +1 +1 +1

3 (f) Determine whether the sequence $200, 180, 160, 140, 120, \ldots$ is geometric, arithmetic, or neither.

$180 - 200 = -20$
$160 - 180 = -20$
$140 - 160 = -20$
$120 - 140 = -20$

so $d = -20$

$a_1 = 200$

Since we are + or − each time its arithmetic +2

+1

$a_n = 200 + (n-1)(-20)$

$a_n = 200 + 20 - 20n$

$\boxed{a_n = -20n + 220}$

* in calc 4 (g) Find the line of best fit for the data:

LinReG

$y = ax + b$

x	1	2	2	3	4
y	2	2	3	4	5

enter data in calculator

Stats → edit → enter x & y values

Quit → Stats → Calc → #4 linReg (ax+b) → enter

$\boxed{y = 1.08x + .6}$ +2 +2

correlation coefficent $r = .94$ since $|.94| \approx 1$ its linear

5 10. Find the equation of the line through the points $(3,7)$ and $(1,3)$.

$x_1\ y_1 \quad x_2\ y_2$

find m

$(3-7)=m(1-3)$

$-4=m(-2)$

$\frac{-4}{-2}=\frac{-2m}{-2}$ +2 $\boxed{m=2}$

+2 $(y-3)=2(x-1)$

$y-3=2x-2$ +1 $\boxed{y=2x+1}$

$+3 \quad +3$

6 11. Find the intercepts of the function $y=\sqrt{x+4}-1$.

to find x intercepts plug in zero for y

to find y intercepts plug in zero for ~~x~~

$y=\sqrt{0+4}-1$ +1

$y=\sqrt{4}-1$

$y=2-1$ +1

$y=1$ $\boxed{(0,1)}$

+1 $0=\sqrt{x+4}-1 \Rightarrow 1^2=\sqrt{x+4}^2$

$1=x+4$

$\boxed{x=-3}$ +1

$\boxed{(-3,0)}$

+1 +1

$\boxed{\text{intercepts @ } (0,1) \text{ \& } (-3,0)}$

9 12. For the rational function $r(x)=\dfrac{3x^2-12}{2x^2-18}$ do the following:

3 (a) Find the domain of $r(x)$.

+1 $2x^2-18 \neq 0$ $\quad \frac{2x^2}{2} \neq \frac{18}{2}$ $\quad x^2 \neq 9$

$\sqrt{x^2} \neq \pm\sqrt{9}$

$x \neq \pm 3$

+1 +1 $\boxed{D: \{x \mid x \neq -3, 3\}}$

3 (b) Find the vertical asymptote(s) of $r(x)$, if any.

+1 $\dfrac{3(x+2)(x-2)}{2(x+3)(x-3)}$ since nothing cancels there are no Holes so $\boxed{VA @ x=-3 \text{ \& } 3}$ +1 +1

3 (c) Find the horizontal asymptote(s) of $r(x)$, if any.

+1 $\frac{P(x)}{Q(x)}$ test

Since the power on top is the same as the power on bottom then we have a $\boxed{HA @ y=\frac{3}{2}}$ +2

6

13. The function $f(x) = x^3 - 1$ is one-to-one. Find the inverse function $f^{-1}(x)$.

(+1) $y = x^3 - 1$

so to find inverse switch x & y then solve for y

(+1) $x = y^3 - 1$

(+1) $y^3 = x + 1$

(+1) $y = \sqrt[3]{x+1}$ Thus (+1) (+1) $\boxed{f^{-1}(x) = \sqrt[3]{x+1}}$

4 14. Solve $\log_4(x + 6) = 2$.

$\log_4(x+6) = 2$ (y 2; b; x)

switch to exponential

(+1) (+1) (+1) $4^2 = (x + 6)$

$16 = x + 6$
$-6 \quad -6$

$\boxed{x = 10}$ (+1)

6 15. Solve $e^{x-4} = 2^{-x}$. Give your answer as an exact value and as a decimal rounded to the nearest hundredth.

★ calc for exact value

(+1) $\ln e^{x-4} = \ln 2^{-x}$

(+1) $x - 4 = \ln 2^{(-x)}$

(+1) $x - 4 = -x(\ln 2)$
$-x \quad -x$

or $x - 4 = -x\ln 2 = x\ln 2 + x = 4$

$x(\ln 2 + 1) = 4$

$x = \dfrac{4}{\ln 2 + 1}$

$-1 + \frac{4}{x} = \ln(2)$ (+1)

$\frac{4}{x} = \ln(2) + 1$

$\frac{x}{4} = \frac{1}{\ln(2)+1}$

exact value (+1) $\boxed{x = \dfrac{4}{\ln(2)+1}}$

decimal (+1) $\boxed{x \approx 2.36}$

4 16. Find the first 5 terms of the sequence defined recursively by $a_1 = 2$, $a_n = 2a_{n-1} + 2$.

$a_1 = \boxed{2}$

$a_2 = 2(a_{2-1}) + 2 = 2(a_1) + 2 = 2(2) + 2 = \boxed{6}$

$a_3 = 2(a_{3-1}) + 2 = 2(6) + 2 = \boxed{14}$

$a_4 = 2(a_{4-1}) + 2 = 2(14) + 2 = \boxed{30}$

$a_5 = 2(30) + 2 = \boxed{62}$

(+1) (+1) (+1) (+1) $\boxed{2, 6, 14, 30, 62}$

8

17. Use elimination or an augmented matrix to solve the system of equations $\left\{\begin{array}{rcrcrcr} -x & & & - & z & = & 2 \\ 2x & + & y & & & = & -5 \\ 2x & & & + & z & = & 0 \end{array}\right\}$.

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & 2 \\ 2 & 1 & 0 & -5 \\ 2 & 0 & 1 & 0 \end{array}\right] \quad R_1 = -R_1 \;\; (+1) \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & -2 \\ 2 & 1 & 0 & -5 \\ 2 & 0 & 1 & 0 \end{array}\right] \quad \begin{array}{l} (+1) \\ R_2 = -2R_1 + R_2 \\ R_3 = -2R_1 + R_3 \\ (+1) \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & -2 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & -1 & 4 \end{array}\right]$$

$$\begin{array}{l} R_3 = -R_3 \\ (+1) \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & -2 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & -4 \end{array}\right] \quad \begin{array}{l} (+1) \\ R_1 = -R_3 + R_1 \\ (+1) \\ R_2 = 2R_3 + R_2 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -4 \end{array}\right]$$

$x = 2$
$y = 9$
$z = -4$

+2

$\boxed{(2, 9, -4)}$

6 18. The matrix $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ represents an augmented matrix for a linear system.

3 (a) Write the corresponding set of linear equations.

(+1) $z = \mathbb{R}$

(+1) $x + z = 7$

(+1) $y + 2z = 2$

$$\boxed{\begin{array}{l} (+1) \\ x = -z + 7 \\ (+1)\; y = -2z + 2 \\ (+1)\; z = \mathbb{R} \end{array}}$$

or $\{x, y, z \mid x = -z + 7,\ y = -2z + 2,\ z = \text{All Reals}\}$

$(-z + 7, -2z + 2, \mathbb{R})$

3 (b) Solve the system.