

Name: \_\_\_\_\_

MATH 1050 - Exam 2 (Released) - 2012-13

NO CALCULATOR

- Neatly write your solutions directly on the exam paper. If a solution requires more space than given, you may continue on the back of the page. Work on scratch paper will not be graded.
- To receive full credit you must show all necessary work and provide clear explanations.
- Books, notes, calculators, and computers are not allowed.
- When you have completed this section, please turn it in to the proctor and get the calculator section of the exam.

1. (6 pts) Find the vertical and horizontal asymptotes (if any) of the function  $h(x) = \frac{x^2 - 2x + 1}{2x^2 - 4x}$ .

\* To Find vertical asymptotes you first need to find the domain restrictions by setting the denominator equal to zero

$$2x^2 - 4x = 2x(x - 2) = 0$$
$$2x = 0 \quad x - 2 = 0$$
$$x = 0 \quad x = 2$$

$$D: \{x \mid x \neq 0, 2\}$$

Now factor the numerator to see if anything will simplify

$$\frac{(x-1)(x-1)}{2x(x-2)}$$

since nothing simplifies we have no holes thus

the remaining domain restrictions are 0, 2 so VA @  $x = 0$  &  $x = 2$

\* since the power of  $m = n$  then we know we have a horizontal asymptote at  $y = \frac{a}{b}$  > leading coefficients  
so we have a H.A. @  $y = \frac{1}{2}$

2. (6 pts) Write  $f(x) = -x^2 - 6x + 2$  in the form  $f(x) = a(x - h)^2 + k$ .

$$(-x^2 - 6x) + 2$$

$$-1(x^2 + 6x) + 2$$

$$-1\left(x^2 + 6x + \left(\frac{6}{2}\right)^2\right) - (-1)\left(\frac{6}{2}\right)^2 + 2$$

$9 + 2 = 11$

$$-1(x + 3)^2 + 11$$

$$y = -(x + 3)^2 + 11$$

Don't forget to multiply by a leading coefficient

remember to add and subtract

$$\left(\frac{b}{2}\right)^2$$

but

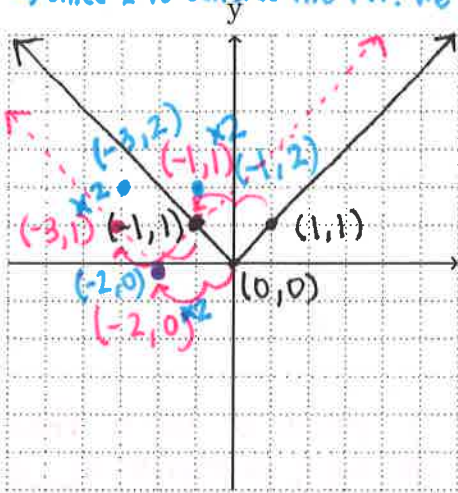
fore you subtract

don't forget to mult by what ever you factored out of your  $x$ 's

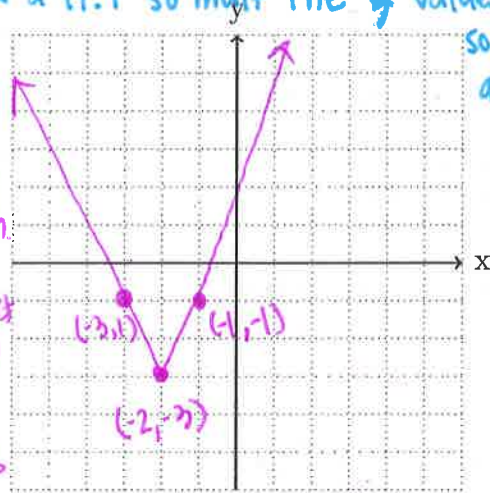
3. (9 pts) For each of the functions below, graph the basic function (for example  $y = x^2$ ). Then use the techniques of shifting, compressing, stretching and/or reflecting to graph each function. Label at least two points on each graph, and any asymptotes. You do NOT need to label the intercepts.

• First we know the Parent Function of  $|x|$  contains pts  $(-1,1), (0,0), (1,1)$   
 PEMDAS  
 Since its inside the Parent fn we know its a vert. trans. & that we do the opposite so  $(-3,1), (-2,0), (-1,1)$   
 Since 2 is outside the P.F. we know its a H.T so mult the values by 2  
 So my new points are  $(-3,2), (-2,0), (-1,2)$

(a)  $g(x) = 2|x+2| - 3$



Since -3 is on the outside of the P.F we know its a Vertical Tran. subtract 3 from all of your y values  
 New points are  $(-3,-1), (-2,-3), (-1,-1)$

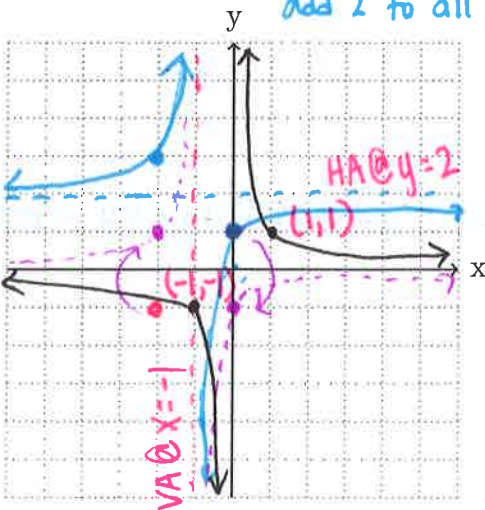


Basic Function  $g(x) = 2|x+2| - 3$

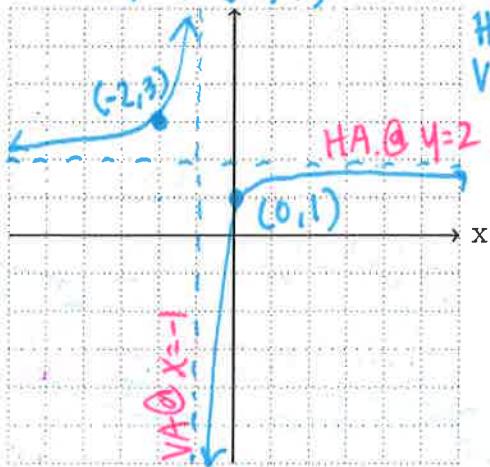
Now graph

• First we know the Parent Function  $P(x) = \frac{1}{x}$  & contains pts  $(-1,-1), (1,1)$   
 Now use order of operations for transformations  
 inside P.F so subtract 1 to all x values  $(-2,-1), (0,1)$   
 negative outside PF mult all y's by -1  $(-2,1), (0,-1)$   
 add 2 to all y values  $(-2,3), (0,1)$

(b)  $f(x) = 2 - \frac{1}{x+1}$



Basic Function

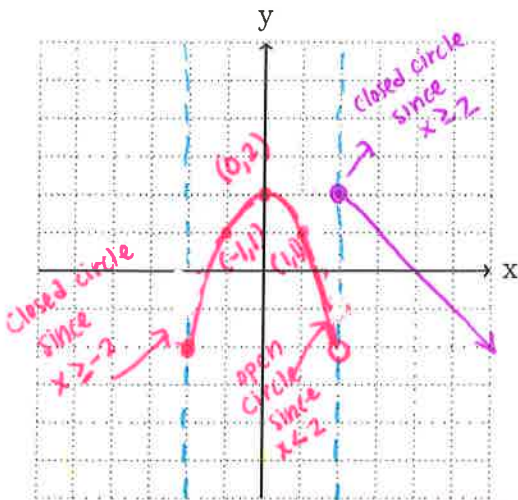


$f(x) = 2 - \frac{1}{x+1}$

HA @  $y=2$   
 VA @  $x=-1$

4. (6 pts) Graph the function  $g(x) = \begin{cases} 2-x^2 & \text{if } -2 \leq x < 2 \\ 4-x & \text{if } x \geq 2 \end{cases}$  look at the domain restrictions

The graph changes between each interval of  $x$



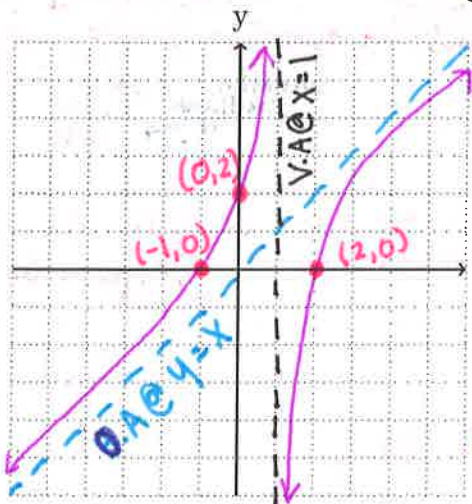
\* graph  $2-x^2$  in the interval  $-2 \leq x < 2$  by first writing it in the correct form  $-x^2 + 2$

neg flips  $+2$   $\uparrow$  2 units remember the domain between  $-2, 2$  but can equal  $-2$

\* graph  $4-x$  in interval  $x \geq 2$   $y = -x + 4$

the neg flips  $+4$   $\uparrow$  4 units

5. (9 pts) Graph  $p(x) = \frac{x^2 - x - 2}{x - 1}$ . Label intercepts and asymptotes.



① D:  $\{x | x \neq 1\}$

②  $\frac{(x+1)(x-2)}{(x-1)}$  since nothing cancels out its in lowest terms & since Nothing canceled that means we have V.A @  $x=1$

③  $(x+1)(x-2) = 0$   $x = -1, 2$   $y = \frac{(0+1)(0-2)}{(0-1)} = \frac{-2}{-1} = 2$   $y = 2$

④ since no holes the remaining domain restrictions is our V.A. @  $x=1$

⑤ since  $M > N$  by a degree of one we will have an O.A. to find it use long division

$$\begin{array}{r} x \rightarrow \text{equation for my O.A. } \boxed{y=x} \\ x-1 \overline{) x^2 - x - 2} \\ \underline{-(x^2 - x)} \phantom{-2} \\ 0 + 0 = -2 \text{ remainder} \end{array}$$

⑥ since we know it will never cross the O.A. then we have enough info to see what our graph is doing

\* Now to see if it crosses oblique

$$\frac{x^2 - x - 2}{x - 1} = x \Rightarrow \frac{(x+1)(x-2)}{(x-1)} = x$$

$(x+1)(x-2) = x(x-1)$  since these will never equal each other they will never intersect the O.A.

end behavior is  $\begin{matrix} \nearrow & \searrow \\ \text{as } x \rightarrow -\infty & y \rightarrow \infty \\ \text{as } x \rightarrow \infty & y \rightarrow -\infty \end{matrix}$

6. (6 pts) Graph the function  $q(x) = -(x-1)^3(x+2)^2$ . Label all intercepts.

① The Power Function is  $f(x) = -x^5$  add  $\nearrow$  neg  $\searrow$  Thus end behavior is

② Intercepts

$$-(x-1)^3(x+2)^2 = 0 \text{ so } x=1, x=-2$$

$$y = -(-0-1)^3(0+2)^2 = -(-1)(4) = 4 \quad y = 4$$

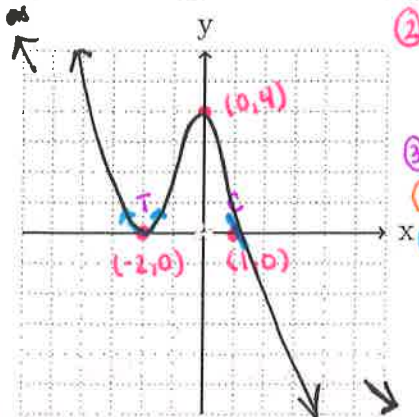
③ since  $x=1$  has a multiplicity of 3 it's odd so it crosses; @  $x=-2$  mult is 2, even, touches

④ max # of turning pts  $n-1 = 5-1 = 4$

⑤  $x \geq 1$   $-(x-1)^3(x+2)^2 = -9(x-1)^3$  since its negative it has a neg slope when it crosses

$$x \leq -2 \quad -(-2-1)^3(x+2)^2 = -(-27)(x+2)^2 = 27(x+2)^2 \text{ since its opens}$$

⑥ graph



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MATH 1050 - Exam 2 (Released) - Fall 2012

Calculator allowed

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- Books, notes, calculators with symbolic manipulation features, computers, cell phones, or other internet enabled devices are not allowed.

1. Each of these problems is worth 3 points. NO justification is required for these problems.

- (a) A degree 4 polynomial  $p(x)$  has zeros  $-1$  (multiplicity 2),  $3$ , and  $2$ . Find a possible equation for  $p(x)$ . Do NOT multiply out your answer.  $a(x+1)^2(x-3)(x-2)$  where  $a$  can be any real #

$$p(x) = (x+1)^2(x-3)(x-2)$$

- (b) Find the line of best fit for the data:

x	1	2	2	3	4
y	1.2	1.3	1.5	1.6	1.8

use calculator STAT → Edit then enter x values under L1 y's under L2  
2nd mode → STAT → CALC → #4 → enter  $y = ax + b$   
 $a = .2$  so  $y = .2x + 1$   
 $b = 1$

- (c) What is the degree of the polynomial  $p(x) = -12x^6 + 8x^3 - 4x$ ?

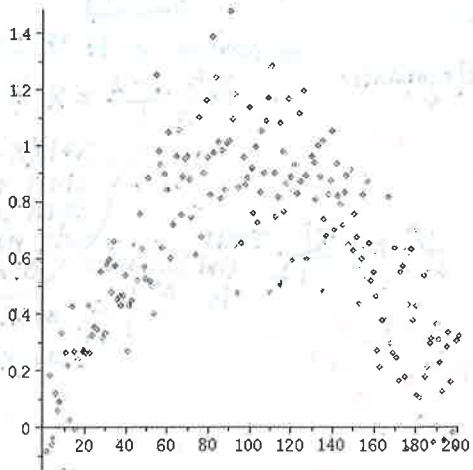
The largest degree is 6 so it Degree of 6

correlation coefficient of  $r^2 = .91$   
 $r = .95$

- (d) Find the axis of symmetry of  $f(x) = 2(x-1)^2 + 4$

Since the vertex is  $(h, k)$  the axis of symmetry is  $h$  so  $X = 1$  is the axis of sym

- (e) Examine the scatter diagram and decide whether the relationship is linear or nonlinear.



non linear since the data doesn't resemble a line

- (f) Janice has a 4ft by 2ft piece of cardboard and wants to make an open top box by cutting a square from each corner and turning up the sides. If  $x$  is length of the side of each square cut out from the box, then the volume of the box is  $V(x) = x(4-2x)(2-2x)$ . For what value of  $x$  is  $V(x)$  largest?

$x(4-2x)(2-2x)$  → graph → 2nd trace → maximum → left bound → right bound → guess

For the value of  $x = .423$  the volume maximizes at 1.5

2. (6 pts) State whether the function  $g(x) = x^3 - 5x + 1$  is odd, even, or neither. (You must justify your answer.)

Since you can use your calculator you can graph it and see if its symmetric with the y-axis (even), origin (odd) or neither

Since the graph is shifted up 1 unit it's **neither**



or you could test even if  $(x,y) \rightarrow (-x,y)$

$$y = (-x)^3 - 5(-x) + 1$$

$$y = -x^3 + 5x + 1 \neq x^3 - 5x + 1$$

NOT EVEN

**Neither**

Odd if  $(x,y) \rightarrow (-x,-y)$

~~$$-y = (-x)^3 - 5(-x) + 1$$~~

$$-y = -x^3 + 5x + 1$$

$$(-y) = -(x^3 - 5x - 1)$$

NOT EVEN SINCE  $x^3 - 5x - 1 \neq x^3 - 5x + 1$

3. (6 pts) A gravel company sells gravel for \$ 10 a ton and charges \$60 for delivery.

- (a) If  $x$  is the number of tons of gravel delivered, write a linear model that relates the cost  $C$ , in dollars, to  $x$ .

Cost Function  $C(x) = 10x + 60$

# of tons (points to  $x$ )

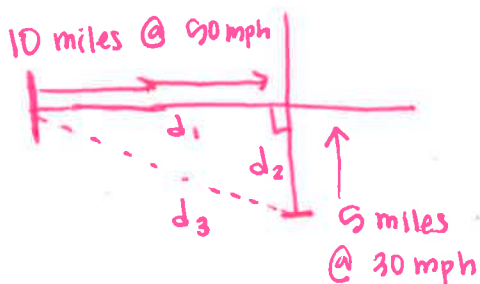
initial cost (constant cost for delivery) (points to 60)

cost per ton (points to 10)

- (b) Find the cost for delivering 8 tons of gravel.

$$C(8) = 10(8) + 60 = \boxed{140}$$

4. (6 pts) Two cars are approaching an intersection. One car is 5 miles south of the intersection and is moving at a constant speed of 30 miles per hour. At the same time, the other car is 10 miles west of the intersection and is moving at a constant speed of 50 miles per hour. Build a model that expresses the distance  $d$  between the cars as a function of time  $t$ .



So we know  $d_1^2 + d_2^2 = d_3^2$

$$d^2 = (10 - 50t)^2 + (5 - 30t)^2$$

$$d = \sqrt{(10 - 50t)^2 + (5 - 30t)^2}$$

$$d = \sqrt{100 - 1000t + 2500t^2 + 25 - 300t + 900t^2}$$

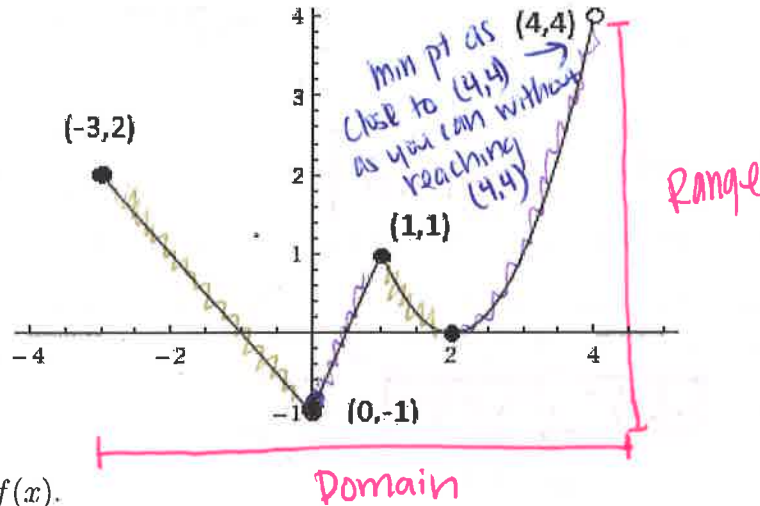
$$d = \sqrt{3400t^2 - 1300t + 125}$$

5. (6 pts) Determine whether  $h(x)$  is linear or non-linear. If  $h(x)$  is linear, find the slope.

$x$	$f(x)$
2	4
3	6
4	9
5	13
6	18

non-linear  
because the slope isn't constant

6. (10 points) For the graph of  $f(x)$ , given below, find the following:



(a) The domain of  $f(x)$ .

$$D: \{x \mid -3 \leq x < 4\} \text{ or } [-3, 4)$$

(b) The range of  $f(x)$ .

$$R: \{y \mid -1 \leq y < 4\} \text{ or } [-1, 4)$$

(c) The interval(s) on which  $f(x)$  is increasing.

increasing on the interval  $(0, 1) \cup (2, 4)$

(d) The interval(s) on which  $f(x)$  is decreasing.

decreasing on the interval  $[-3, 0) \cup (1, 2)$

(e) The absolute maximum and minimum, if they exist.

absolute maximum point at  $(3.9, 3.9)$

absolute minimum point at  $(0, -1)$

Objective- tested	Likelihood	Section	Comments
Use a given graph to find intercepts, domain, range, odd, even, neither, local maximum, local minimum, and to find intervals where increasing/decreasing	Probably	Calc	
Use a given graph to find absolute maximum, absolute minimum	Maybe	Calc	
Determine algebraically whether each function is even, odd, or neither	Probably	No Calc	
Be able to determine local maximum, local minimum and intervals where the function is increasing/decreasing by using a graphing calculator	Maybe	Calc	More likely to give students the graph
3.4 Be able to graph piecewise defined functions	Definitely	No Calc	
Memorize functions in the library of functions (not the greatest integer function, $\text{int}(x)$ )	Definitely	No Calc	
Be able to evaluate a piecewise defined function at a point	Maybe	Either	
3.5 Find the equation of a function given as a transformation of a library function	Maybe	No Calc	
Be able to use transformations and knowledge of library functions to graph functions without a calculator.	Definitely	No Calc	
Given a graph of $f(x)$ be able to graph related functions by transformations	Maybe	Either	
3.6 Be able to set up maximization problems.	Definitely	Calc	Just 7,18, 19, 22, 23
Be able to use a calculator to solve a maximization problem	Probably	Calc	Either from a problem you set up, or from an equation that you are given.
4.1 Be familiar with terminology average rate of change and slope for linear functions	Maybe	Calc	
Be able to tell whether a function is linear given a table of points	Probably	Calc	
Be able to create linear models and answer questions using a linear model.	Definitely	Calc	See 4.1: 37, 38, 42, 47-50 (9th edition) OR 37, 38, 44, 49-52 (8th edition). There will be no supply and demand problems, and no straight line depreciation problems.
4.2 Be able to draw a scatterplot diagram	Maybe	Calc	
Be able to examine a scatter diagram and determine whether the relationship is linear or non-linear	Probably	Calc	
Be able to use a graphing utility to find the line of best fit.	Definitely	Calc	
4.3 Given a quadratic function $f(x)=ax^2+bx+c$ , be able to write it in the standard form $f(x)=a(x-h)^2+k$ (without a calculator)	Probably	No Calc	
Be able to graph a quadratic function using transformations (without a calculator)	Maybe	No Calc	One of many functions in the library of functions that you could be asked to graph
For a quadratic function, be able to find the vertex, axis of symmetry, intercepts, domain, range, and intervals where the function is increasing/decreasing	Probably	Calc	
4.4 Not tested			
Know the terminology associated with polynomials: polynomial function, power function, degree, real zero, multiplicity, repeated zero, touches, crosses, turning point, end behavior			Need these to be able to understand the questions that are being asked.
5.1 Memorize the power functions as part of the library of functions. Be able to use transformations of $x^4$ or $x^5$ to graph	Maybe	No Calc	One of many possible functions

		Probably	Calc	Will likely show up either with real zeros or complex zeros
Be able to use information about real zeros and degree to form a polynomial function		Probably	Calc	
Given a graph of a polynomial function, be able to find an equation for the polynomial		Maybe	Calc	
Given a polynomial, be able to determine (1) the zeros and their multiplicity, (2) the x- and y- intercepts, (3) the end behavior, (4) the maximum number of turning points, and (4) whether the graph touches or crosses at each zero without using a calculator		Probably	No Calc	
Given a factored polynomial, be able to sketch a graph of the polynomial using the steps of this section		Definitely	No Calc	Will just grade final graph for correctness, not looking for a list of all the steps. Also, I will not give a function with factors like $x^2+4$ .
5.2 Be able to find the domain of a rational function.		Probably	Calc	
Know what vertical, horizontal, and oblique asymptotes are. Be able to find them from a graph.		Maybe	No Calc	
Know how to find the vertical, horizontal, and oblique asymptotes of a rational function without a calculator.		Probably	No Calc	
Memorize the graphs of the functions $f(x)=1/x$ and $g(x)=1/x^2$ as part of your library of functions			No Calc	
Be able to use transformations of $1/x$ , $1/x^2$ to graph rational functions without a calculator.		Maybe	No Calc	One of many
5.3 Be able to graph a rational function, labeling intercepts and asymptotes. *		Definitely	Calc	* See below
Given the graph of a rational function with vertical and horizontal asymptotes, be able to find a possible equation for the function		Maybe	Calc	
* For graphing rational functions, there will be no functions with holes or quadratic end behavior (see 5.3:13, 14). All graphs will have either a horizontal or oblique asymptote.				