24. $f(x)=3x^2-2x+4$

$\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{3(x+h)^2-2(x+h)+4-(3x^2-2x+4)}{h}$

$=\dfrac{3(x^2+2xh+h^2)-2x-2h+4-3x^2+2x-4}{h}$

$=\dfrac{3x^2+6xh+3h^2-2x-2h+4-3x^2+2x-4}{h}$

$=\dfrac{6xh+3h^2-2h}{h}=\dfrac{h(6x+3h-2)}{h}$

$=6x+3h-2$

25. a. Domain: $\{x\mid -4\le x\le 3\}$; $[-4,3]$

Range: $\{y\mid -3\le y\le 3\}$; $[-3,3]$

b. Intercept: $(0,0)$

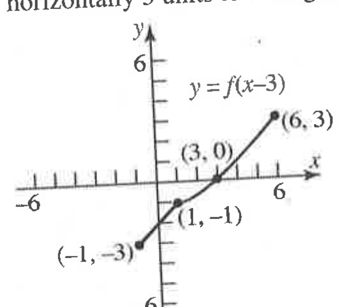
c. $f(-2)=-1$

d. $f(x)=-3$ when $x=-4$

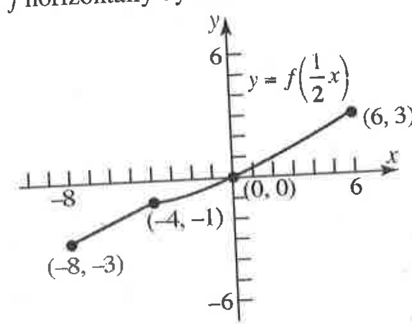
e. $f(x)>0$ when $0<x\le 3$

$\{x\mid 0<x\le 3\}$

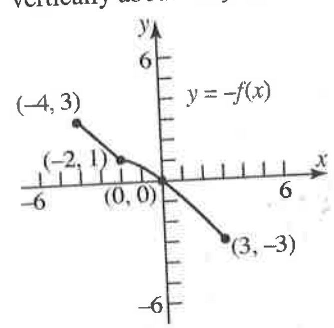
f. To graph $y=f(x-3)$, shift the graph of f horizontally 3 units to the right.



g. To graph $y=f\left(\frac{1}{2}x\right)$, stretch the graph of f horizontally by a factor of 2.



h. To graph $y=-f(x)$, reflect the graph of f vertically about the y-axis.



26. a. Domain: $\{x\mid -5\le x\le 4\}$; $[-5,4]$

Range: $\{y\mid -3\le y\le 1\}$; $[-3,1]$

b. $g(-1)=1$

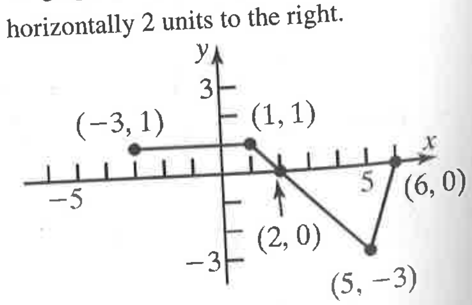
c. Intercepts: $(0,0)$, $(4,0)$

d. $g(x)=-3$ when $x=3$

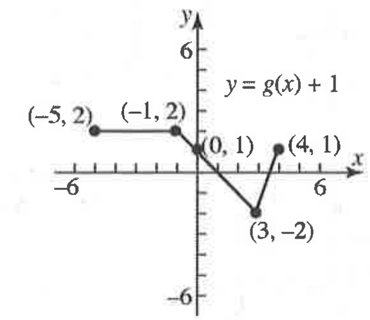
e. $g(x)>0$ when $-5\le x<0$

$\{x\mid -5\le x<0\}$

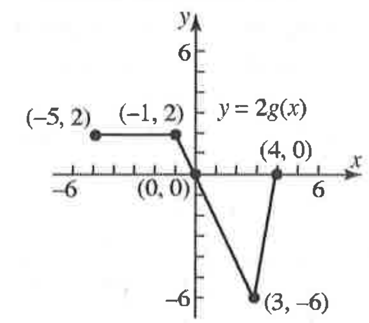
f. To graph $y=g(x-2)$, shift the graph of g horizontally 2 units to the right.



g. To graph $y=g(x)+1$, shift the graph of g vertically up 1 unit.



h. To graph $y=2g(x)$, stretch the graph of g vertically by a factor of 2.



27. a. Domain: $\{x\mid -4\le x\le 4\}$; $[-4,4]$

Range: $\{y\mid -3\le y\le 1\}$; $[-3,1]$

b. Increasing: $(-4,-1)$ and $(3,4)$;

Decreasing: $(-1,3)$

c. Local minimum is -3 when $x=3$;
Local maximum is 1 when $x=-1$.
Note that $x=4$ and $x=-4$ do not yield local extrema because there is no open interval that contains either value.

d. Absolute minimum is -3 when $x=3$;
Absolute maximum is 1 when $x=-1$.

e. The graph is not symmetric with respect to the x-axis, the y-axis or the origin.

f. The function is neither even nor odd.

g. x-intercepts: $-2,0,4$
y-intercept: 0

28. a. Domain: $\{x\mid x \text{ is any real number}\}$

Range: $\{y\mid y \text{ is any real number}\}$

b. Increasing: $(-\infty,-2)$ and $(2,\infty)$;

Decreasing: $(-2,2)$

c. Local minimum is -1 at $x=2$;
Local maximum is 1 at $x=-2$

d. Absolute minimum is -3 at $x=-4$;
Absolute maximum is 3 at $x=4$

e. The graph is symmetric with respect to the origin.

f. The function is odd.

g. x-intercepts: $-3,0,3$;
y-intercept: 0

29. $f(x)=x^3-4x$

$f(-x)=(-x)^3-4(-x)=-x^3+4x$

$=-(x^3-4x)=-f(x)$

f is odd.

30. $g(x)=\dfrac{4+x^2}{1+x^4}$

$g(-x)=\dfrac{4+(-x)^2}{1+(-x)^4}=\dfrac{4+x^2}{1+x^4}=g(x)$

g is even.

31. $h(x)=\dfrac{1}{x^4}+\dfrac{1}{x^2}+1$

$h(-x)=\dfrac{1}{(-x)^4}+\dfrac{1}{(-x)^2}+1=\dfrac{1}{x^4}+\dfrac{1}{x^2}+1=h(x)$

h is even.

32. $F(x)=\sqrt{1-x^3}$

$F(-x)=\sqrt{1-(-x)^3}=\sqrt{1+x^3}\ne F(x)$ or $-F(x)$

F is neither even nor odd.

33. $G(x)=1-x+x^3$

$G(-x)=1-(-x)+(-x)^3$

$=1+x-x^3\ne -G(x)$ or $G(x)$

G is neither even nor odd.

34. $H(x)=1+x+x^2$

$H(-x)=1+(-x)+(-x)^2$

$=1-x+x^2\ne -H(x)$ or $H(x)$

H is neither even nor odd.

35. $f(x)=\dfrac{x}{1+x^2}$

$f(-x)=\dfrac{-x}{1+(-x)^2}=\dfrac{-x}{1+x^2}=-f(x)$

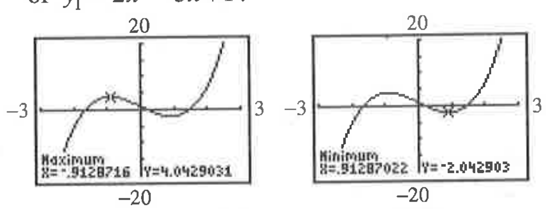
f is odd.

36. $g(x)=\dfrac{1+x^2}{x^3}$

$g(-x)=\dfrac{1+(-x)^2}{(-x)^3}=\dfrac{1+x^2}{-x^3}=-\dfrac{1+x^2}{x^3}=-g(x)$

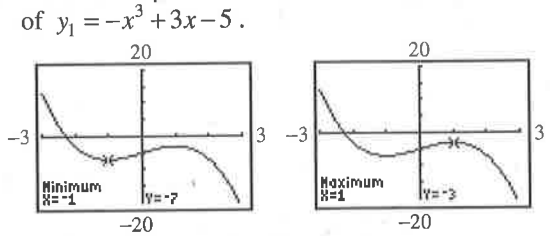
g is odd.

37. $f(x)=2x^3-5x+1$ on the interval $(-3,3)$
Use MAXIMUM and MINIMUM on the graph of $y_1=2x^3-5x+1$.



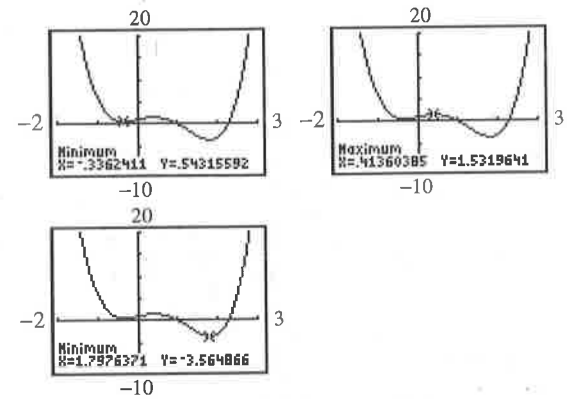
local maximum: 4.04 when $x\approx -0.91$
local minimum: -2.04 when $x=0.91$
f is increasing on: $(-3,-0.91)$ and $(0.91,3)$;
f is decreasing on: $(-0.91,0.91)$.

38. $f(x)=-x^3+3x-5$ on the interval $(-3,3)$
Use MAXIMUM and MINIMUM on the graph of $y_1=-x^3+3x-5$.



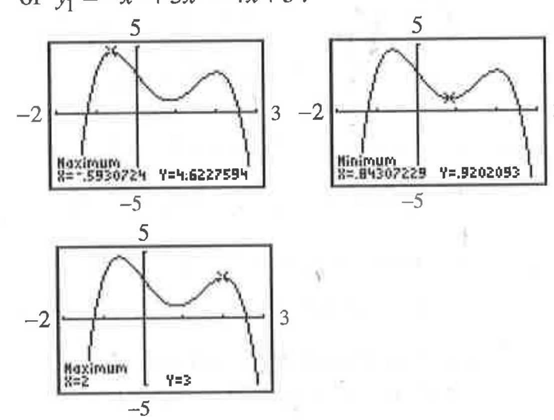
local maximum: -3 when $x=1$
local minimum: -7 when $x=-1$
f is increasing on: $(-1,1)$;
f is decreasing on: $(-3,-1)$ and $(1,3)$.

39. $f(x)=2x^4-5x^3+2x+1$ on the interval $(-2,3)$
Use MAXIMUM and MINIMUM on the graph of $y_1=2x^4-5x^3+2x+1$.



local maximum: 1.53 when $x=0.41$
local minima: 0.54 when $x=-0.34$, -3.56 when $x=1.80$
f is increasing on: $(-0.34,0.41)$ and $(1.80,3)$;
f is decreasing on: $(-2,-0.34)$ and $(0.41,1.80)$.

40. $f(x)=-x^4+3x^3-4x+3$ on the interval $(-2,3)$
Use MAXIMUM and MINIMUM on the graph of $y_1=-x^4+3x^3-4x+3$.



local maximum: 4.62 when $x=-0.59$, 3 when $x=2$
local minimum: 0.92 when $x=0.84$
f is increasing on: $(-2,-0.59)$ and $(0.84,2)$;
f is decreasing on: $(-0.59,0.84)$ and $(2,3)$.

41. $f(x)=8x^2-x$

a. $\dfrac{f(2)-f(1)}{2-1}=\dfrac{8(2)^2-2-[8(1)^2-1]}{1}=32-2-(7)=23$

b. $\dfrac{f(1)-f(0)}{1-0}=\dfrac{8(1)^2-1-[8(0)^2-0]}{1}=8-1-(0)=7$

c. $\dfrac{f(4)-f(2)}{4-2}=\dfrac{8(4)^2-4-[8(2)^2-2]}{2}=\dfrac{128-4-(30)}{2}=\dfrac{94}{2}=47$

42. $f(x)=2x^3+x$

a. $\dfrac{f(2)-f(1)}{2-1}=\dfrac{2(2)^3+2-\left(2(1)^3+1\right)}{1}=16+2-(3)=15$

b. $\dfrac{f(1)-f(0)}{1-0}=\dfrac{2(1)^3+1-\left(2(0)^3+0\right)}{1}=2+1-(0)=3$

c. $\dfrac{f(4)-f(2)}{4-2}=\dfrac{2(4)^3+4-(2(2)^3+2)}{2}=\dfrac{128+4-(18)}{2}=\dfrac{114}{2}=57$

43. $f(x)=2-5x$

$\dfrac{f(3)-f(2)}{3-2}=\dfrac{[2-5(3)]-[2-5(2)]}{3-2}=\dfrac{(2-15)-(2-10)}{1}=-13-(-8)=-5$

44. $f(x)=2x^2+7$

$\dfrac{f(3)-f(2)}{3-2}=\dfrac{\left[2(3)^2+7\right]-\left[2(2)^2+7\right]}{3-2}=\dfrac{(18+7)-(8+7)}{1}=25-15=10$

45. $f(x)=3x-4x^2$

$\dfrac{f(3)-f(2)}{3-2}=\dfrac{\left[3(3)-4(3)^2\right]-\left[3(2)-4(2)^2\right]}{3-2}=\dfrac{(9-36)-(6-16)}{1}=-27+10=-17$

46. $f(x)=x^2-3x+2$

$\dfrac{f(3)-f(2)}{3-2}=\dfrac{\left[(3)^2-3(3)+2\right]-\left[(2)^2-3(2)+2\right]}{3-2}=\dfrac{(9-9+2)-(4-6+2)}{1}=2-0=2$

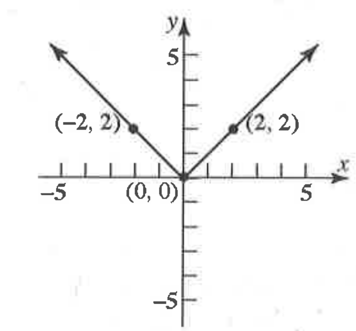
47. The graph does not pass the Vertical Line Test and is therefore not a function.

48. The graph passes the Vertical Line Test and is therefore a function.

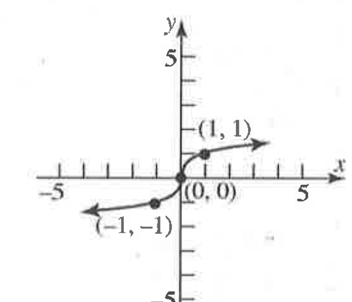
49. The graph passes the Vertical Line Test and is therefore a function.

50. The graph passes the Vertical Line Test and is therefore a function.

51. $f(x)=|x|$



52. $f(x)=\sqrt[3]{x}$



Total Area = $\text{area}_{\text{square}} + \text{area}_{\text{circle}} = x^2 + \pi r^2$

$$A(x) = x^2 + \pi\left(\frac{5-2x}{\pi}\right)^2 = x^2 + \frac{25-20x+4x^2}{\pi}$$

b. Since the lengths must be positive, we have:

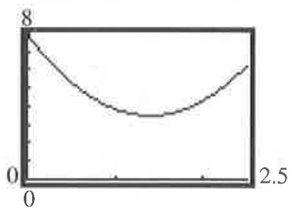
$10-4x>0$ and $x>0$

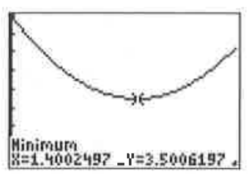
$-4x>-10$ and $x>0$

$x<2.5$ and $x>0$

Domain: $\{x \mid 0<x<2.5\}$

c. The total area is smallest when $x \approx 1.40$ meters.





12. a. C = circumference, A = total area, r = radius, x = side of equilateral triangle

$$C = 2\pi r = 10-3x \Rightarrow r = \frac{10-3x}{2\pi}$$

The height of the equilateral triangle is $\frac{\sqrt{3}}{2}x$.

Total Area = $\text{area}_{\text{triangle}} + \text{area}_{\text{circle}}$

$$= \frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right) + \pi r^2$$

$$A(x) = \frac{\sqrt{3}}{4}x^2 + \pi\left(\frac{10-3x}{2\pi}\right)^2$$

$$= \frac{\sqrt{3}}{4}x^2 + \frac{100-60x+9x^2}{4\pi}$$

b. Since the lengths must be positive, we have:

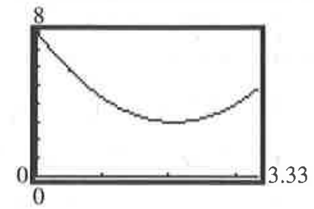
$10-3x>0$ and $x>0$

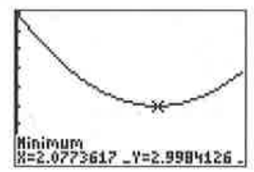
$-3x>-10$ and $x>0$

$x<\frac{10}{3}$ and $x>0$

Domain: $\left\{x \,\middle|\, 0<x<\frac{10}{3}\right\}$

c. The area is smallest when $x \approx 2.08$ meters.





13. a. Since the wire of length x is bent into a circle, the circumference is x. Therefore, $C(x) = x$.

b. Since $C = x = 2\pi r$, $r = \frac{x}{2\pi}$.

$$A(x) = \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}.$$

14. a. Since the wire of length x is bent into a square, the perimeter is x. Therefore, $p(x) = x$.

b. Since $P = x = 4s$, $s = \frac{1}{4}x$, we have

$$A(x) = s^2 = \left(\frac{1}{4}x\right)^2 = \frac{1}{16}x^2.$$

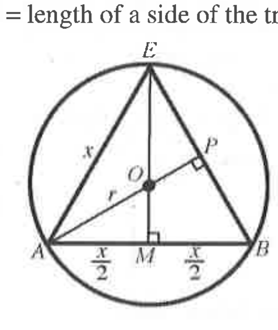
15. a. A = area, r = radius; diameter = $2r$

$A(r) = (2r)(r) = 2r^2$

b. p = perimeter

$p(r) = 2(2r) + 2r = 6r$

16. C = circumference, r = radius; x = length of a side of the triangle



Since ΔABC is equilateral, $EM = \frac{\sqrt{3}x}{2}$.

Therefore, $OM = \frac{\sqrt{3}x}{2} - OE = \frac{\sqrt{3}x}{2} - r$

In ΔOAM, $r^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2} - r\right)^2$

$$r^2 = \frac{x^2}{4} + \frac{3}{4}x^2 - \sqrt{3}rx + r^2$$

$$\sqrt{3}rx = x^2$$

$$r = \frac{x}{\sqrt{3}}$$

Therefore, the circumference of the circle is

$$C(x) = 2\pi r = 2\pi\left(\frac{x}{\sqrt{3}}\right) = \frac{2\pi\sqrt{3}}{3}x$$

17. Area of the equilateral triangle

$$A = \frac{1}{2}x \cdot \frac{\sqrt{3}}{2}x = \frac{\sqrt{3}}{4}x^2$$

From problem 16, we have $r^2 = \frac{x^2}{3}$.

Area inside the circle, but outside the triangle:

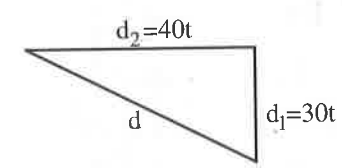
$$A(x) = \pi r^2 - \frac{\sqrt{3}}{4}x^2$$

$$= \pi\frac{x^2}{3} - \frac{\sqrt{3}}{4}x^2 = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)x^2$$

18. $d^2 = d_1^2 + d_2^2$

$$d^2 = (30t)^2 + (40t)^2$$

$$d(t) = \sqrt{900t^2 + 1600t^2} = \sqrt{2500t^2} = 50t$$



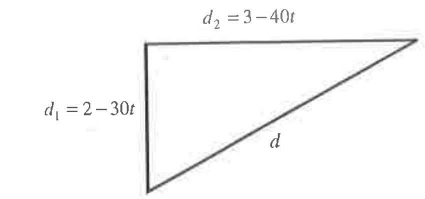
19. a. $d^2 = d_1^2 + d_2^2$

$$d^2 = (2-30t)^2 + (3-40t)^2$$

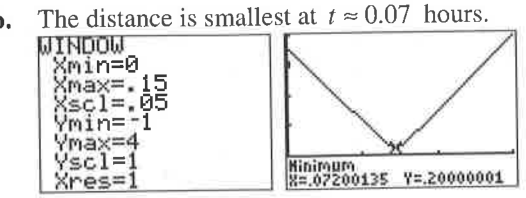
$$d(t) = \sqrt{(2-30t)^2 + (3-40t)^2}$$

$$= \sqrt{4 - 120t + 900t^2 + 9 - 240t + 1600t^2}$$

$$= \sqrt{2500t^2 - 360t + 13}$$



b. The distance is smallest at $t \approx 0.07$ hours.



20. r = radius of cylinder, h = height of cylinder, V = volume of cylinder

$$r^2 + \left(\frac{h}{2}\right)^2 = R^2 \Rightarrow r^2 + \frac{h^2}{4} = R^2 \Rightarrow r^2 = R^2 - \frac{h^2}{4}$$

$$V = \pi r^2 h$$

$$V(h) = \pi\left(R^2 - \frac{h^2}{4}\right)h = \pi h\left(R^2 - \frac{h^2}{4}\right)$$

21. r = radius of cylinder, h = height of cylinder, V = volume of cylinder

By similar triangles: $\frac{H}{R} = \frac{H-h}{r}$

$$Hr = R(H-h)$$

$$Hr = RH - Rh$$

$$Rh = RH - Hr$$

$$h = \frac{RH - Hr}{R} = \frac{H(R-r)}{R}$$

$$V = \pi r^2 h = \pi r^2\left(\frac{H(R-r)}{R}\right) = \frac{\pi H(R-r)r^2}{R}$$

22. a. The total cost of installing the cable along the road is $500x$. If cable is installed x miles along the road, there are $5-x$ miles between the road to the house and where the cable ends along the road.

7.

x	$y=f(x)$	Avg. rate of change $=\frac{\Delta y}{\Delta x}$
-1	-2	
0	3	$\frac{3-(-2)}{0-(-1)}=\frac{5}{1}=5$
1	8	$\frac{8-3}{1-0}=\frac{5}{1}=5$
2	13	$\frac{13-8}{2-1}=\frac{5}{1}=5$
3	18	$\frac{18-13}{3-2}=\frac{5}{1}=5$

This is a linear function with slope = 5, since the average rate of change is constant at 5.

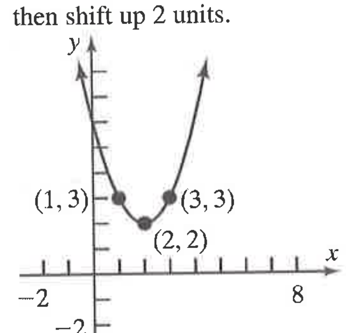
8.

x	$y=f(x)$	Avg. rate of change $=\frac{\Delta y}{\Delta x}$
-1	-3	
0	4	$\frac{4-(-3)}{0-(-1)}=\frac{7}{1}=7$
1	7	$\frac{7-4}{1-0}=\frac{3}{1}=3$
2	6	
3	1	

This is not a linear function, since the average rate of change is not constant.

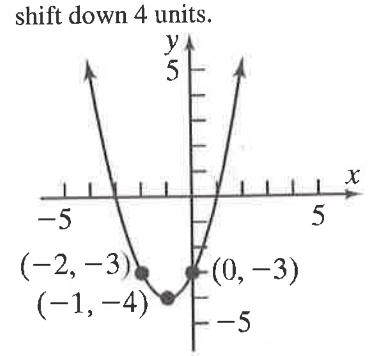
9. $f(x)=(x-2)^2+2$

Using the graph of $y=x^2$, shift right 2 units, then shift up 2 units.



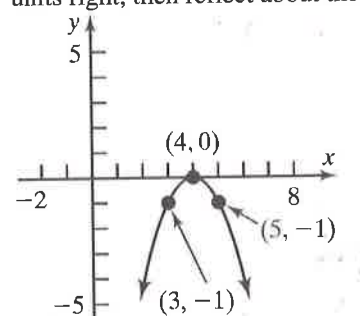
10. $f(x)=(x+1)^2-4$

Using the graph of $y=x^2$, shift left 1 unit, then shift down 4 units.



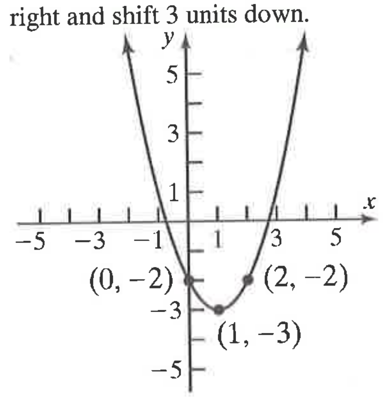
11. $f(x)=-(x-4)^2$

Using the graph of $y=x^2$, shift the graph 4 units right, then reflect about the x-axis.



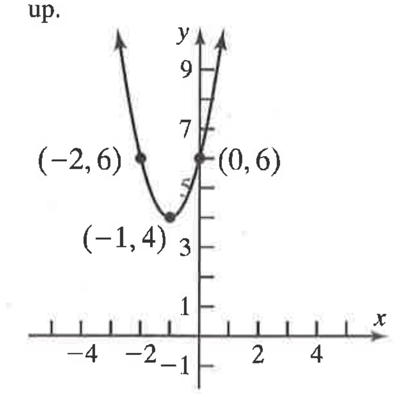
12. $f(x)=(x-1)^2-3$

Using the graph of $y=x^2$, shift the graph 1 unit right and shift 3 units down.



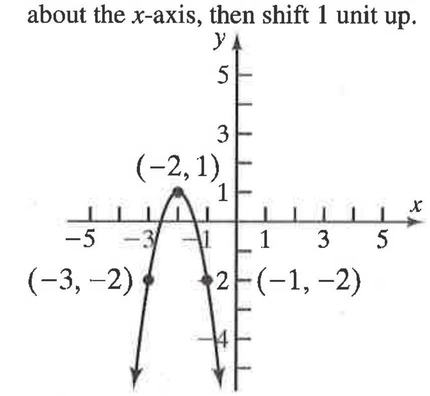
13. $f(x)=2(x+1)^2+4$

Using the graph of $y=x^2$, stretch vertically by a factor of 2, then shift 1 unit left, then shift 4 units up.



14. $f(x)=-3(x+2)^2+1$

Using the graph of $y=x^2$, stretch vertically by a factor of 3, then shift 2 units left, then reflect about the x-axis, then shift 1 unit up.



15. a. $f(x)=(x-2)^2+2$

$=x^2-4x+4+2$

$=x^2-4x+6$

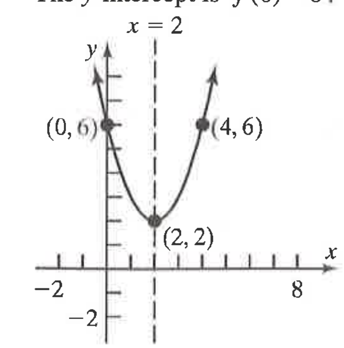
$a=1, b=-4, c=6$. Since $a=1>0$, the graph opens up. The x-coordinate of the vertex is $x=-\frac{b}{2a}=-\frac{-4}{2(1)}=\frac{4}{2}=2$.

The y-coordinate of the vertex is

$f\left(-\frac{b}{2a}\right)=f(2)=(2)^2-4(2)+6=2$.

Thus, the vertex is (2, 2).
The axis of symmetry is the line $x=2$.
The discriminant is:
$b^2-4ac=(-4)^2-4(1)(6)=-8<0$, so the graph has no x-intercepts.

The y-intercept is $f(0)=6$.



b. The domain is $(-\infty,\infty)$.
The range is $[2,\infty)$.

c. Decreasing on $(-\infty,2)$.
Increasing on $(2,\infty)$.

16. a. $f(x)=(x+1)^2-4$

$=x^2+2x+1-4$

$=x^2+2x-3$

$a=1, b=2, c=2$. Since $a=1>0$, the graph opens up. The x-coordinate of the vertex is $x=-\frac{b}{2a}=-\frac{2}{2(1)}=-1$.

The y-coordinate of the vertex is

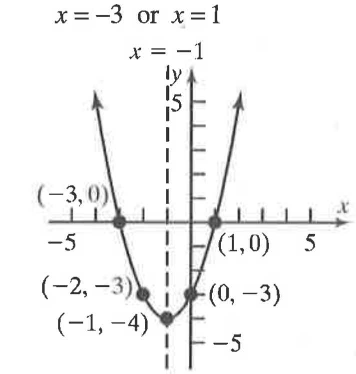
$f\left(-\frac{b}{2a}\right)=f(-1)=(-1)^2+2(-1)-3=-4$.

Thus, the vertex is $(-1,-4)$.
The axis of symmetry is the line $x=-1$.
The discriminant is:
$b^2-4ac=(2)^2-4(1)(-3)=16>0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3$ or $x=1$



26. $f(x) = 2x^2 + 8x + 5$
$a = 2, b = 8, c = 5$. Since $a = 2 > 0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at

$$x = -\frac{b}{2a} = -\frac{8}{2(2)} = -\frac{8}{4} = -2.$$

The minimum value is

$$f\left(-\frac{b}{2a}\right) = f(-2) = 2(-2)^2 + 8(-2) + 5$$
$$= 8 - 16 + 5 = -3$$

27. $f(x) = -x^2 + 8x - 4$
$a = -1, b = 8, c = -4$. Since $a = -1 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at

$$x = -\frac{b}{2a} = -\frac{8}{2(-1)} = -\frac{8}{-2} = 4.$$

The maximum value is

$$f\left(-\frac{b}{2a}\right) = f(4) = -(4)^2 + 8(4) - 4$$
$$= -16 + 32 - 4 = 12$$

28. $f(x) = -x^2 - 10x - 3$
$a = -1, b = -10, c = -3$. Since $a = -1 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at

$$x = -\frac{b}{2a} = -\frac{-10}{2(-1)} = \frac{10}{-2} = -5.$$

The maximum value is

$$f\left(-\frac{b}{2a}\right) = f(-5) = -(-5)^2 - 10(-5) - 3$$
$$= -25 + 50 - 3 = 22$$

29. $f(x) = -3x^2 + 12x + 4$
$a = -3, b = 12, c = 4$. Since $a = -3 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at

$$x = -\frac{b}{2a} = -\frac{12}{2(-3)} = -\frac{12}{-6} = 2.$$

The maximum value is

$$f\left(-\frac{b}{2a}\right) = f(2) = -3(2)^2 + 12(2) + 4$$
$$= -12 + 24 + 4 = 16$$

30. $f(x) = -2x^2 + 4$
$a = -2, b = 0, c = 4$. Since $a = -2 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at

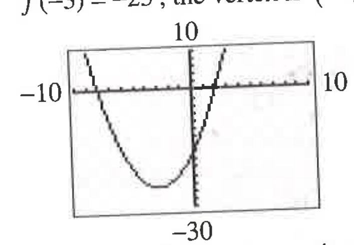
$$x = -\frac{b}{2a} = -\frac{0}{2(-2)} = 0.$$

The maximum value is

$$f\left(-\frac{b}{2a}\right) = f(0) = -2(0)^2 + 4 = 4.$$

31. $x^2 + 6x - 16 < 0$
We graph the function $f(x) = x^2 + 6x - 16$. The intercepts are
y-intercept: $f(0) = -16$
x-intercepts: $x^2 + 6x - 16 = 0$
$(x+8)(x-2) = 0$
$x = -8, x = 2$

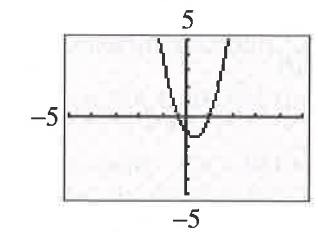
The vertex is at $x = \frac{-b}{2a} = \frac{-(6)}{2(1)} = -3$. Since $f(-3) = -25$, the vertex is $(-3, -25)$.



The graph is below the x-axis when $-8 < x < 2$. Since the inequality is strict, the solution set is $\{x \mid -8 < x < 2\}$ or, using interval notation, $(-8, 2)$.

32. $3x^2 - 2x - 1 \ge 0$
We graph the function $f(x) = 3x^2 - 2x - 1$. The intercepts are
y-intercept: $f(0) = -1$
x-intercepts: $3x^2 - 2x - 1 = 0$
$(3x+1)(x-1) = 0$
$x = -\frac{1}{3}, x = 1$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-2)}{2(3)} = \frac{2}{6} = \frac{1}{3}$. Since $f\left(\frac{1}{3}\right) = -\frac{4}{3}$, the vertex is $\left(\frac{1}{3}, -\frac{4}{3}\right)$.

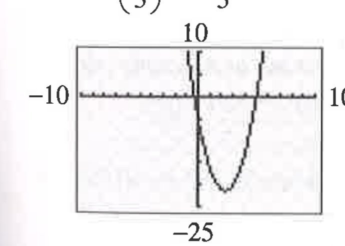


The graph is above the x-axis when $x < -\frac{1}{3}$ or $x > 1$. Since the inequality is not strict, the solution set is $\left\{x \middle| x \le -\frac{1}{3} \text{ or } x \ge 1\right\}$ or, using interval notation, $\left(-\infty, -\frac{1}{3}\right] \cup [1, \infty)$.

33. $3x^2 \ge 14x + 5$
$3x^2 - 14x - 5 \ge 0$
We graph the function $f(x) = 3x^2 - 14x - 5$. The intercepts are
y-intercept: $f(0) = -5$
x-intercepts: $3x^2 - 14x - 5 = 0$
$(3x+1)(x-5) = 0$
$x = -\frac{1}{3}, x = 5$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-14)}{2(3)} = \frac{14}{6} = \frac{7}{3}$.

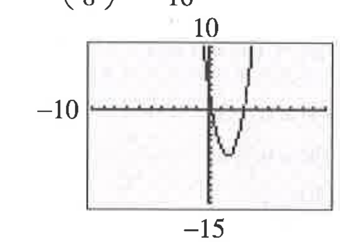
Since $f\left(\frac{7}{3}\right) = -\frac{64}{3}$, the vertex is $\left(\frac{7}{3}, -\frac{64}{3}\right)$.



The graph is above the x-axis when $x < -\frac{1}{3}$ or $x > 5$. Since the inequality is not strict, the solution set is $\left\{x \middle| x \le -\frac{1}{3} \text{ or } x \ge 5\right\}$ or, using interval notation, $\left(-\infty, -\frac{1}{3}\right] \cup [5, \infty)$.

34. $4x^2 < 13x - 3$
$4x^2 - 13x + 3 < 0$
We graph the function $f(x) = 4x^2 - 13x + 3$. The intercepts are
y-intercept: $f(0) = 3$
x-intercepts: $4x^2 - 13x + 3 = 0$
$(4x-1)(x-3) = 0$
$x = \frac{1}{4}, x = 3$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-13)}{2(4)} = \frac{13}{8}$. Since $f\left(\frac{13}{8}\right) = -\frac{121}{16}$, the vertex is $\left(\frac{13}{8}, -\frac{121}{16}\right)$.



The graph is below the x-axis when $\frac{1}{4} < x < 3$. Since the inequality is strict, the solution set is $\left\{x \middle| \frac{1}{4} < x < 3\right\}$ or, using interval notation, $\left(\frac{1}{4}, 3\right)$.

35. Use the form $f(x) = a(x-h)^2 + k$.
The vertex is $(-1, 2)$, so $h = -1$ and $k = 2$.
$f(x) = a(x+1)^2 + 2$.
Since the graph passes through $(1, 6)$, $f(1) = 6$.
$6 = a(1+1)^2 + 2$
$6 = a(2)^2 + 2$
$6 = 4a + 2$
$4 = 4a$
$1 = a$
$f(x) = 1(x+1)^2 + 2$
$= (x^2 + 2x + 1) + 2$
$= x^2 + 2x + 3$

36. Use the form $f(x) = a(x-h)^2 + k$.
The vertex is $(3, -4)$, so $h = 3$ and $k = -4$.
$f(x) = a(x-3)^2 - 4$.
Since the graph passes through $(4, 2)$, $f(4) = 2$.

$2 = a(4-3)^2 - 4$
$2 = a(1)^2 - 4$
$2 = a - 4$
$6 = a$
$f(x) = 6(x-3)^2 - 4$
$= 6(x^2 - 6x + 9) - 4$
$= 6x^2 - 36x + 54 - 4$
$= 6x^2 - 36x + 50$

37. a. Company A: $C(x) = 0.06x + 7.00$
Company B: $C(x) = 0.08x$

b. $0.06x + 7.00 = 0.08x$
$7.00 = 0.02x$
$350 = x$
The bill from Company A will equal the bill from Company B if 350 minutes are used.

c. $0.08x < 0.06x + 7.00$
$0.02x < 7.00$
$x < 350$
The bill from Company B will be less than the bill from Company A if fewer than 350 minutes are used. That is, $0 \le x < 350$.

38. a. $S(x) = 0.01x + 15{,}000$

b. $S(1{,}000{,}000) = 0.01(1{,}000{,}000) + 15{,}000$
$= 10{,}000 + 15{,}000 = 25{,}000$
In 2005, Bill's salary was \$25,000.

c. $0.01x + 15{,}000 = 100{,}000$
$0.01x = 85{,}000$
$x = 8{,}500{,}000$
Bill's sales would have to be \$8,500,000 in order to earn \$100,000.

d. $0.01x + 15{,}000 > 150{,}000$
$0.01x > 135{,}000$
$x > 13{,}500{,}000$
Bill's sales would have to be more than \$13,500,000 in order for his salary to exceed \$150,000.

39. a. The revenue will equal the quantity x sold times the price p. That is, $R = xp$. Thus,

$$R(x) = x\left(-\frac{1}{10}x + 150\right) = -\frac{1}{10}x^2 + 150x$$

b. $R(100) = -\frac{1}{10}(100)^2 + 150(100) = 14{,}000$
The revenue is \$14,000 if 100 units are sold.

c. $a = -\frac{1}{10}, b = 150, c = 0$. Since $a = -\frac{1}{10} < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \frac{-b}{2a} = \frac{-(150)}{2(-1/10)} = \frac{-150}{-1/5} = 750$. Thus, the quantity that maximizes revenue is 750 units. The maximum revenue is

$$R(750) = -\frac{1}{10}(750)^2 + 150(750)$$
$$= -56{,}250 + 112{,}500$$
$$= \$56{,}250$$

d. From part (c), we know revenue is maximizes when $x = 750$ units are sold. The price that should be charged for this is

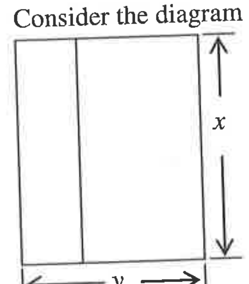
$$p = -\frac{1}{10}(750) + 150 = \$75.$$

40. Since there are 200 feet of border, we know that $2x + 2y = 200$. The area is to be maximized, so $A = x \cdot y$. Solving the perimeter formula for y:
$2x + 2y = 200$
$2y = 200 - 2x$
$y = 100 - x$
The area function is:
$A(x) = x(100 - x) = -x^2 + 100x$
The maximum value occurs at the vertex:

$$x = \frac{-b}{2a} = \frac{-(100)}{2(-1)} = \frac{-100}{-2} = 50$$

The pond should be 50 feet by 50 feet for maximum area.

41. Consider the diagram



Total amount of fence $= 3x + 2y = 10{,}000$

$$y = \frac{10{,}000 - 3x}{2} = 5000 - \frac{3}{2}x$$

Total area enclosed $= (x)(y) = (x)\left(5000 - \frac{3}{2}x\right)$

$A(x) = 5000x - \frac{3}{2}x^2 = -\frac{3}{2}x^2 + 5000x$ is a quadratic function with $a = -\frac{3}{2} < 0$.

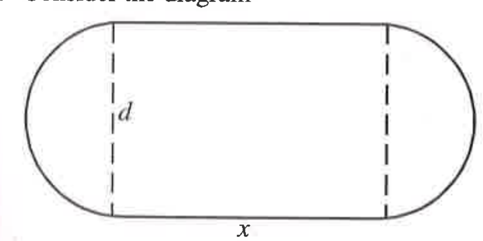
So the vertex corresponds to the maximum value for this function. The vertex occurs when

$$x = -\frac{b}{2a} = -\frac{5000}{2(-3/2)} = \frac{5000}{3}.$$

The maximum area is:

$$A\left(\frac{5000}{3}\right) = -\frac{3}{2}\left(\frac{5000}{3}\right)^2 + 5000\left(\frac{5000}{3}\right)$$
$$= -\frac{3}{2}\left(\frac{25{,}000{,}000}{9}\right) + \frac{25{,}000{,}000}{3}$$
$$= -\frac{12{,}500{,}000}{3} + \frac{25{,}000{,}000}{3}$$
$$= \frac{12{,}500{,}000}{3}$$
$$\approx 4{,}166{,}666.67 \text{ square meters}$$

42. Consider the diagram



Let d = diameter of the semicircles
= width of the rectangle
Let x = length of the rectangle
100 = outside dimension length
$100 = 2x + 2(\text{circumference of a semicircle})$
$100 = 2x + \text{circumference of a circle}$
$100 = 2x + \pi d$
$100 - \pi d = 2x$
$\frac{100 - \pi d}{2} = x$
$50 - \frac{1}{2}\pi d = x$

We need an expression for the area of a rectangle in terms of a single variable.
$A_{\text{rectangle}} = x \cdot d$
$= \left(50 - \frac{1}{2}\pi d\right) \cdot d$
$= 50d - \frac{1}{2}\pi d^2$

This is a quadratic function with $a = -\frac{1}{2}\pi < 0$.
Therefore, the x-coordinate of the vertex represents the value for d that maximizes the area of the rectangle and the y-coordinate of the vertex is the maximum area of the rectangle. The vertex occurs at

$$d = -\frac{b}{2a} = -\frac{50}{2\left(-\frac{1}{2}\pi\right)} = \frac{-50}{-\pi} = \frac{50}{\pi}$$

This gives us

$$x = 50 - \frac{1}{2}\pi d = 50 - \frac{1}{2}\pi\left(\frac{50}{\pi}\right) = 50 - 25 = 25$$

Therefore, the side of the rectangle with the semicircle should be $\frac{50}{\pi}$ feet and the other side should be 25 feet. The maximum area is $(25)\left(\frac{50}{\pi}\right) = \frac{1250}{\pi} \approx 397.89 \text{ ft}^2$.

43. $C(x) = 4.9x^2 - 617.4x + 19{,}600$;
$a = 4.9, b = -617.4, c = 19{,}600$. Since $a = 4.9 > 0$, the graph opens up, so the vertex is a minimum point.

a. The minimum marginal cost occurs at
$$x = -\frac{b}{2a} = -\frac{-617.40}{2(4.9)} = \frac{617.40}{9.8} = 63.$$
Thus, 63 golf clubs should be manufactured in order to minimize the marginal cost.

b. The minimum marginal cost is
$C(63) = 4.9(63)^2 - (617.40)(63) + 19600$
$= \$151.90$

44. The area function is:
$A(x) = x(10 - x) = -x^2 + 10x$
The maximum value occurs at the vertex:

$$x = -\frac{b}{2a} = -\frac{10}{2(-1)} = -\frac{10}{-2} = 5$$

The maximum area is:

Chapter 5 Review Exercises

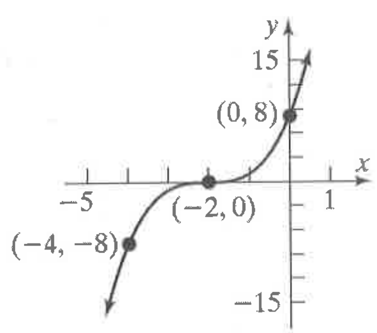
1. $f(x)=4x^5-3x^2+5x-2$ is a polynomial of degree 5.

2. $f(x)=\dfrac{3x^5}{2x+1}$ is a rational function. It is not a polynomial because there are variables in the denominator.

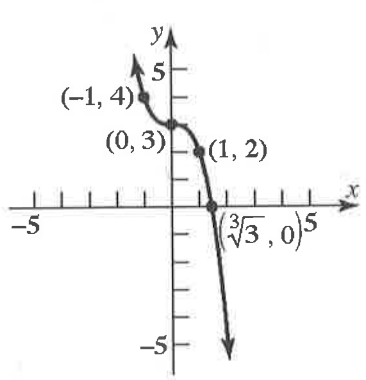
3. $f(x)=3x^2+5x^{1/2}-1$ is not a polynomial because the variable x is raised to the $\frac{1}{2}$ power, which is not a nonnegative integer.

4. $f(x)=3$ is a polynomial of degree 0.

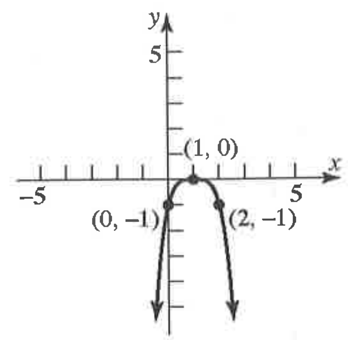
5. $f(x)=(x+2)^3$
Using the graph of $y=x^3$, shift left 2 units.



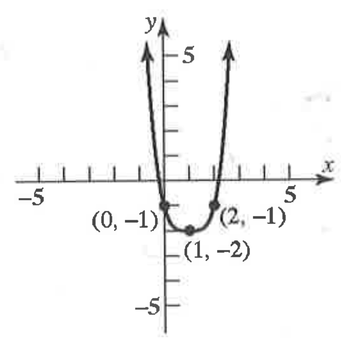
6. $f(x)=-x^3+3$
Using the graph of $y=x^3$, reflect about the x-axis, then shift up 3 units.



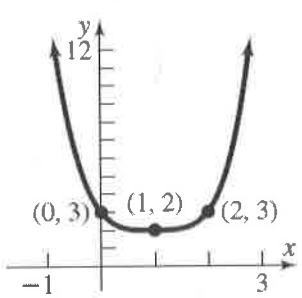
7. $f(x)=-(x-1)^4$
Using the graph of $y=x^4$, shift right 1 unit, then reflect about the x-axis.



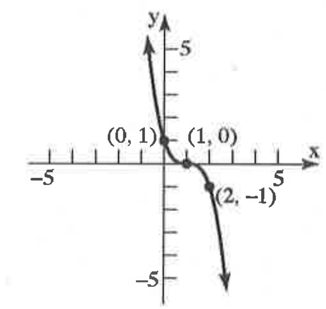
8. $f(x)=(x-1)^4-2$
Using the graph of $y=x^4$, shift right 1 unit, then shift down 2 units.



9. $f(x)=(x-1)^4+2$
Using the graph of $y=x^4$, shift right 1 unit, then shift up 2 units.



10. $f(x)=(1-x)^3=-(x-1)^3$
Using the graph of $y=x^3$, shift right 1 unit, then reflect about the x-axis.



11. $f(x)=x(x+2)(x+4)$

Step 1: Degree is 3. The function resembles $y=x^3$ for large values of $|x|$.

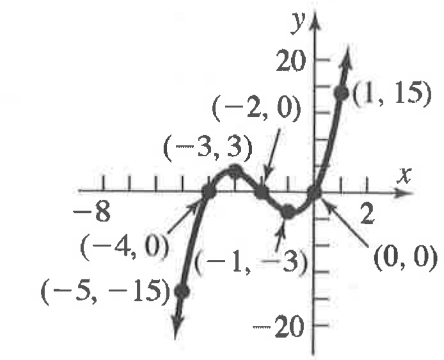
Step 2: y-intercept: $f(0)=(0)(0+2)(0+4)=0$
x-intercepts: solve $f(x)=0$:
$x(x+2)(x+4)=0$
$x=0$ or $x=-2$ or $x=-4$

Step 3: The graph crosses the x-axis at $x=-4$, $x=-2$ and $x=0$ since each zero has multiplicity 1.

Step 4: The polynomial is of degree 3 so the graph has at most $3-1=2$ turning points.

Step 5: Near -4:
$f(x)\approx-4(-4+2)(x+4)=8(x+4)$
(a line with slope 8)
Near -2:
$f(x)\approx-2(x+2)(-2+4)=-4(x+2)$
(a line with slope -4)
Near 0: $f(x)\approx x(0+2)(0+4)=8x$
(a line with slope 8)

Step 6: Graphing:



12. $f(x)=x(x-2)(x-4)$

Step 1: Degree is 3. The function resembles $y=x^3$ for large values of $|x|$.

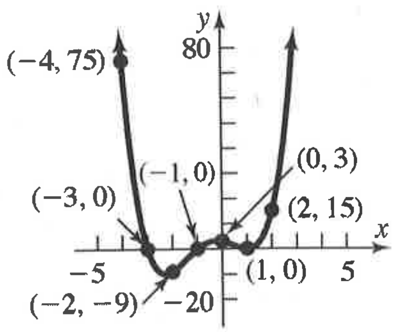
Step 2: y-intercept: $f(0)=(0)(0-2)(0-4)=0$
x-intercepts: solve $f(x)=0$:
$x(x-2)(x-4)=0$
$x=0$ or $x=2$ or $x=4$

Step 3: The graph crosses the x-axis at $x=0$, $x=2$ and $x=4$ since each zero has multiplicity 1.

Step 4: The polynomial is of degree 3 so the graph has at most $3-1=2$ turning points.

Step 5: Near 0: $f(x)\approx x(0-2)(0-4)=8x$
(a line with slope 8)
Near 2:
$f(x)\approx2(x-2)(2-4)=-4(x-2)$
(a line with slope -4)
Near 4:
$f(x)\approx4(4-2)(x-4)=8(x-4)$
(a line with slope 8)

Step 6: Graphing:



18. $f(x)=(x-4)(x+2)^2(x-2)$

Step 1: Degree is 4. The function resembles $y=x^4$ for large values of $|x|$.

Step 2: y-intercept:
$f(0)=(0-4)(0+2)^2(0-2)$
$=32$
x-intercepts: solve $f(x)=0$:
$(x-4)(x+2)^2(x-2)=0$
$x=4$ or $x=-2$ or $x=2$

Step 3: The graph crosses the x-axis at $x=2$ and $x=4$ since each zero has multiplicity 1. The graph touches the x-axis at $x=-2$ since this zero has multiplicity 2.

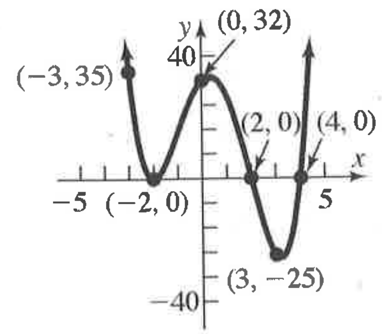
Step 4: The polynomial is of degree 4 so the graph has at most $4-1=3$ turning points.

Step 5: Near -2:
$f(x)\approx(-2-4)(x+2)^2(-2-2)=24(x+2)^2$
(a parabola opening upward)

Near 2:
$f(x)\approx(2-4)(2+2)^2(x-2)=-32(x-2)$
(a line with slope -32)

Near 4:
$f(x)\approx(x-4)(4+2)^2(4-2)=72(x-4)$
(a line with slope 72)

Step 6: Graphing:



19. $R(x)=\dfrac{x+2}{x^2-9}=\dfrac{x+2}{(x+3)(x-3)}$ is in lowest terms. The denominator has zeros at -3 and 3. Thus, the domain is $\{x\mid x\neq-3, x\neq 3\}$. The degree of the numerator, $p(x)=x+2$, is $n=1$. The degree of the denominator $q(x)=x^2-9$, is $m=2$. Since $n<m$, the line $y=0$ is a horizontal asymptote. Since the denominator is zero at -3 and 3, $x=-3$ and $x=3$ are vertical asymptotes.

20. $R(x)=\dfrac{x^2+4}{x-2}$ is in lowest terms. The denominator has a zero at 2. Thus, the domain is $\{x\mid x\neq 2\}$. The degree of the numerator, $p(x)=x^2+4$, is $n=2$. The degree of the denominator, $q(x)=x-2$, is $m=1$. Since $n=m+1$, there is an oblique asymptote.

Dividing:

$$\begin{array}{r|l} & x+2 \\ x-2 & x^2+4 \\ & x^2-2x \\ & 2x+4 \\ & 2x-4 \\ & 8 \end{array}$$

$$R(x)=x+2+\frac{8}{x-2}$$

Thus, the oblique asymptote is $y=x+2$. Since the denominator is zero at 2, $x=2$ is a vertical asymptote.

21. $R(x)=\dfrac{x^2+3x+2}{(x+2)^2}=\dfrac{(x+2)(x+1)}{(x+2)^2}=\dfrac{x+1}{x+2}$ is in lowest terms. The denominator has a zero at -2. Thus, the domain is $\{x\mid x\neq-2\}$. The degree of the numerator, $p(x)=x^2+3x+2$, is $n=2$. The degree of the denominator, $q(x)=(x+2)^2=x^2+4x+4$, is $m=2$. Since $n=m$, the line $y=\frac{1}{1}=1$ is a horizontal asymptote. Since the denominator of $y=\dfrac{x+1}{x+2}$ is zero at -2, $x=-2$ is a vertical asymptote.

22. $R(x)=\dfrac{x^3}{x^3-1}$ is in lowest terms. The denominator has a zero at 1. Thus, the domain is $\{x\mid x\neq 1\}$. The degree of the numerator, $p(x)=x^3$, is $n=3$. The degree of the denominator, $q(x)=x^3-1$, is $m=3$. Since $n=m$, the line $y=\frac{1}{1}=1$ is a horizontal asymptote. Since the denominator is zero at 1, $x=1$ is a vertical asymptote.

23. $R(x)=\dfrac{2x-6}{x}$ $p(x)=2x-6$; $q(x)=x$; $n=1$; $m=1$

Step 1: Domain: $\{x\mid x\neq 0\}$
There is no y-intercept because 0 is not in the domain.

Step 2: $R(x)=\dfrac{2x-6}{x}=\dfrac{2(x-3)}{x}$ is in lowest terms.

Step 3: The x-intercept is the zero of $p(x)$: 3
Near 3: $R(x)\approx\frac{2}{3}(x-3)$. Plot the point $(3,0)$ and show a line with positive slope there.

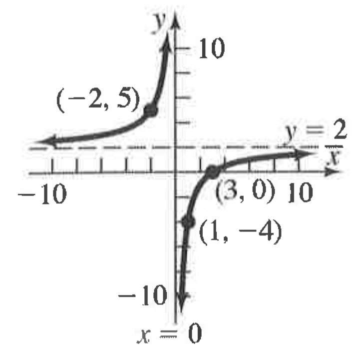
Step 4: $R(x)=\dfrac{2x-6}{x}=\dfrac{2(x-3)}{x}$ is in lowest terms. The vertical asymptote is the zero of $q(x)$: $x=0$.
Graph this asymptote using a dashed line.

Step 5: Since $n=m$, the line $y=\frac{2}{1}=2$ is the horizontal asymptote. Solve to find intersection points:
$\dfrac{2x-6}{x}=2$
$2x-6=2x$
$-6\neq 0$
$R(x)$ does not intersect $y=2$. Plot the line $y=2$ with dashes.

Step 6:

Interval	$(-\infty, 0)$	$(0, 3)$	$(3, \infty)$
Number Chosen	-2	1	4
Value of R	$R(-2)=5$	$R(1)=-4$	$R(4)=\frac{1}{2}$
Location of Graph	Above x-axis	Below x-axis	Above x-axis
Point on Graph	$(-2, 5)$	$(1, -4)$	$(4, \frac{1}{2})$

Steps 7 & 8: Graphing:



24. $R(x)=\dfrac{4-x}{x}$ $p(x)=4-x;\ q(x)=x;\ n=1;\ m=1$

Step 1: Domain: $\{x \mid x \neq 0\}$

Step 2: $R(x)=\dfrac{4-x}{x}$ is in lowest terms.

Step 3: There is no y-intercept because 0 is not in the domain.

The x-intercept is the zero of $p(x)$: 4

Near 4: $R(x)\approx\dfrac{4-x}{4}=-\dfrac{1}{4}x+1$. Plot the point $(4,0)$ and show a line with negative slope there.

Step 4: $R(x)=\dfrac{4-x}{x}$ is in lowest terms. The vertical asymptote is the zero of $q(x)$: $x=0$.

Graph this asymptote using a dashed line.

Step 5: Since $n=m$, the line $y=\dfrac{-1}{1}=-1$ is the horizontal asymptote. Solve to find intersection points:

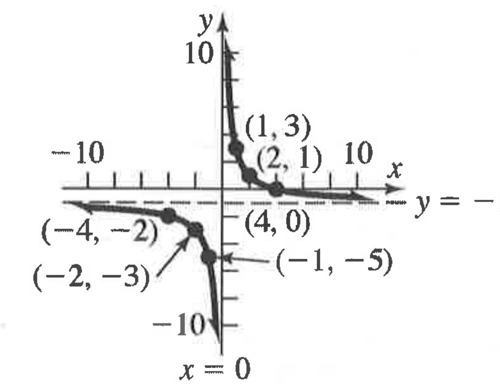
$$\frac{4-x}{x}=-1$$
$$4-x=-x$$
$$4\neq 0$$

$R(x)$ does not intersect $y=-1$. Plot the line $y=-1$ using dashes.

Step 6:

Interval	$(-\infty, 0)$	$(0, 4)$	$(4, \infty)$
Number Chosen	-1	1	5
Value of R	$R(-1)=-5$	$R(1)=3$	$R(5)=-0.2$
Location of Graph	Below x-axis	Above x-axis	Below x-axis
Point on Graph	$(-1,-5)$	$(1,3)$	$(5,-0.2)$

Steps 7 & 8: Graphing:



25. $H(x)=\dfrac{x+2}{x(x-2)}$ $p(x)=x+2;\ q(x)=x(x-2)=x^2-2x;\ n=1;\ m=2$

Step 1: Domain: $\{x \mid x\neq 0, x\neq 2\}$.

Step 2: $H(x)=\dfrac{x+2}{x(x-2)}$ is in lowest terms.

Step 3: There is no y-intercept because 0 is not in the domain.

The x-intercept is the zero of $p(x)$: -2

Near -2: $H(x)\approx\dfrac{1}{8}(x+2)$. Plot the point $(-2,0)$ and show a line with positive slope there.

Step 4: $H(x)=\dfrac{x+2}{x(x-2)}$ is in lowest terms. The vertical asymptotes are the zeros of $q(x)$: $x=0$ and $x=2$.

Graph these asymptotes using dashed lines.

Step 5: Since $n<m$, the line $y=0$ is the horizontal asymptote. Solve to find intersection points:

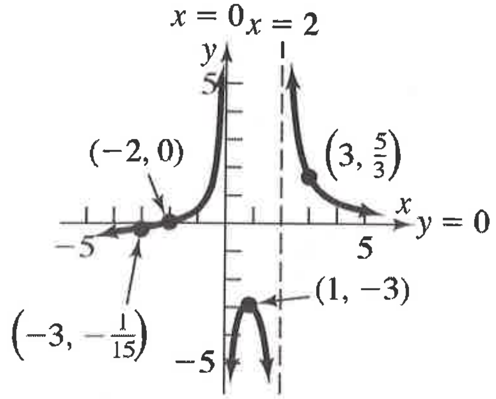
$$\frac{x+2}{x(x-2)}=0$$
$$x+2=0$$
$$x=-2$$

$H(x)$ intersects $y=0$ at $(-2, 0)$. Plot the line $y=0$ using dashes.

Step 6:

Interval	$(-\infty,-2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
Number Chosen	-3	-1	1	3
Value of H	$H(-3)=-\frac{1}{15}$	$H(-1)=\frac{1}{3}$	$H(1)=-3$	$H(3)=\frac{5}{3}$
Location of Graph	Below x-axis	Above x-axis	Below x-axis	Above x-axis
Point on Graph	$\left(-3,-\frac{1}{15}\right)$	$\left(-1,\frac{1}{3}\right)$	$(1,-3)$	$\left(3,\frac{5}{3}\right)$

Steps 7 & 8: Graphing:



26. $H(x)=\dfrac{x}{x^2-1}=\dfrac{x}{(x-1)(x+1)}$ $p(x)=x;\ q(x)=x^2-1;\ n=1;\ m=2$

Step 1: Domain: $\{x \mid x\neq -1, x\neq 1\}$.

Step 2: $H(x)=\dfrac{x}{x^2-1}=\dfrac{x}{(x-1)(x+1)}$ is in lowest terms.

Step 3: The y-intercept is $H(0)=\dfrac{0}{0^2-1}=0$. Plot the point $(0,0)$.

The x-intercept is the zero of $p(x)$: 0

Near 0: $H(x)\approx -x$. Plot the point $(0,0)$ and show a line with negative slope there.

Step 4: $H(x)=\dfrac{x}{x^2-1}=\dfrac{x}{(x-1)(x+1)}$ is in lowest terms. The vertical asymptotes are the zeros of $q(x)$: $x=-1$ and $x=1$. Graph these asymptotes using dashed lines.

Step 5: Since $n<m$, the line $y=0$ is the horizontal asymptote. Solve to find intersection points:

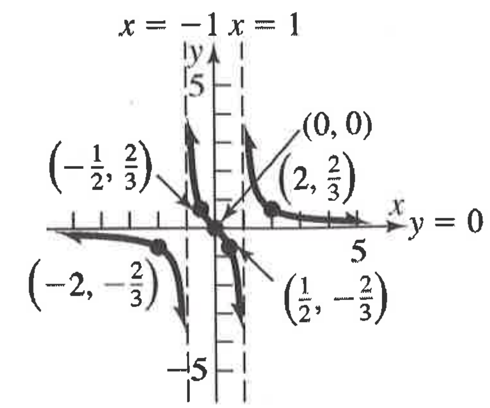
$$\frac{x}{x^2-1}=0$$
$$x=0$$

$H(x)$ intersects $y=0$ at (0, 0). Plot the line $y=0$ using dashes.

Step 6:

Interval	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
Number Chosen	-2	$-\frac{1}{2}$	$\frac{1}{2}$	2
Value of H	$H(-2)=-\frac{2}{3}$	$H\left(-\frac{1}{2}\right)=\frac{2}{3}$	$H\left(\frac{1}{2}\right)=-\frac{2}{3}$	$H(2)=\frac{2}{3}$
Location of Graph	Below x-axis	Above x-axis	Below x-axis	Above x-axis
Point on Graph	$\left(-2,-\frac{2}{3}\right)$	$\left(-\frac{1}{2},\frac{2}{3}\right)$	$\left(\frac{1}{2},-\frac{2}{3}\right)$	$\left(2,\frac{2}{3}\right)$

Steps 7 & 8: Graphing:



27. $R(x)=\dfrac{x^2+x-6}{x^2-x-6}=\dfrac{(x+3)(x-2)}{(x-3)(x+2)}$ $p(x)=x^2+x-6;\ q(x)=x^2-x-6;$

Step 1: Domain: $\{x \mid x\neq -2, x\neq 3\}$.

Step 2: $R(x)=\dfrac{x^2+x-6}{x^2-x-6}$ is in lowest terms.

Step 3: The y-intercept is $R(0)=\dfrac{0^2+0-6}{0^2-0-6}=\dfrac{-6}{-6}=1$. Plot the point $(0,1)$.

The x-intercepts are the zeros of $p(x)$: -3 and 2.

Near -3: $R(x)\approx -\frac{5}{6}(x+3)$. Plot the point $(-3,0)$ and show a line with negative slope there.

Near 2: $R(x)\approx -\frac{5}{4}(x-2)$. Plot the point $(2,0)$ and show a line with negative slope there.

Step 4: $R(x)=\dfrac{x^2+x-6}{x^2-x-6}$ is in lowest terms. The vertical asymptotes are the zeros of $q(x)$: $x=-2$ and $x=3$. Graph these asymptotes with dashed lines.

Step 5: Since $n=m$, the line $y=\frac{1}{1}=1$ is the horizontal asymptote. Solve to find intersection points:

$$\frac{x^2+x-6}{x^2-x-6}=1$$
$$x^2+x-6=x^2-x-6$$
$$2x=0$$
$$x=0$$

$R(x)$ intersects $y=1$ at (0, 1). Plot the line $y=1$ using dashes.

Step 6: